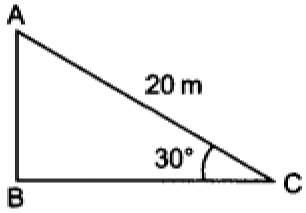
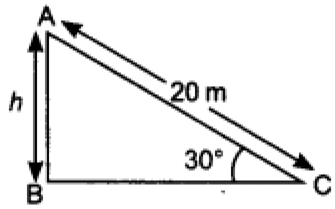
NCERT Solutions Of Chapter 9 – Some Applications Of Trigonometry

Ex 9.1

Question 1.
A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure).



Solution:
Given: length of the rope (AC) = 20 m, ∠ACB = 30°
Let height of the pole (AB) = h metres



In ΔABC, $\frac{AB}{AC} = \sin 30°$

$\Rightarrow \frac{h}{20} = \frac{1}{2} \Rightarrow h = \frac{20}{2} = 10 \text{ m}$

Hence, height of the pole = 10 m

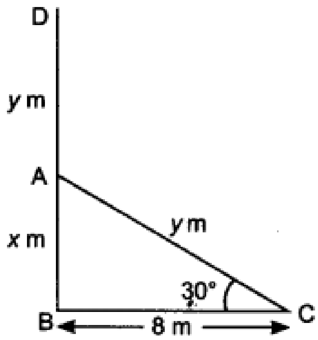
Question 2.
A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.
Solution:

Let DB is a tree and AD is the broken part of it which touches the ground at C.

Given: $\angle ACB = 30°$ and $BC = 8$ m

Let $AB = x$ m and $AD = y$ m

$\therefore$ Now, length of the tree $= (x + y)$ m



In ΔABC,

$$\frac{AB}{BC} = \tan 30° \Rightarrow \frac{x}{8} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{8}{\sqrt{3}} \quad ...(i)$$

and $$\frac{AB}{AC} = \sin 30° \Rightarrow \frac{x}{y} = \frac{1}{2}$$

$$\Rightarrow y = 2x \Rightarrow y = 2 \times \frac{8}{\sqrt{3}} = \frac{16}{\sqrt{3}}$$ [From equation (i)]

Hence, total height of the tree

$$x + y = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8 \times 1.732 = 13.856 \text{ m}$$

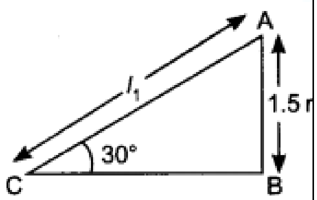
Question 3.

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Solution:

Let l_1 is the length of slide for children below the age of 5 years and l_2 is the length of the slide for elder children

for elder children



In ΔABC, $$\frac{AB}{AC} = \sin 30° \Rightarrow \frac{1.5}{AC} = \frac{1}{2}$$

$$\Rightarrow AC = 3 \text{ m} \Rightarrow l_1 = 3 \text{ m}$$

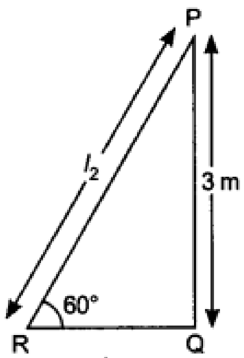
In ΔPQR, $$\frac{PQ}{PR} = \sin 60°$$

$$\Rightarrow \frac{3}{PR} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow PR = \frac{3 \times 2}{\sqrt{3}}$$

$$= \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ m}$$

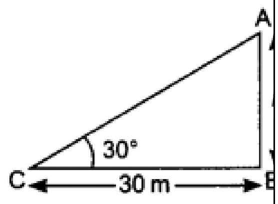
$$l_2 = 2\sqrt{3} \text{ m}$$



Question 4.
The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.
Solution:
Let h be the height of the tower
In ΔABC,

In ΔABC, $\frac{AB}{BC} = \tan 30° \Rightarrow \frac{h}{30} = \frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$ m



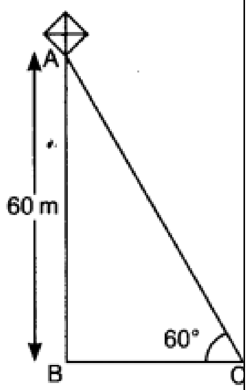
Question 5.
A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.
Solution:
Given: height AB = 60 m, ∠ACB = 60°, AC = length of the string

In ΔABC, $\frac{AB}{AC} = \sin 60°$

$\Rightarrow \frac{60}{AC} = \frac{\sqrt{3}}{2}$

$\Rightarrow AC = \frac{60 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}$ m

Hence, length of the string = $40\sqrt{3}$ m



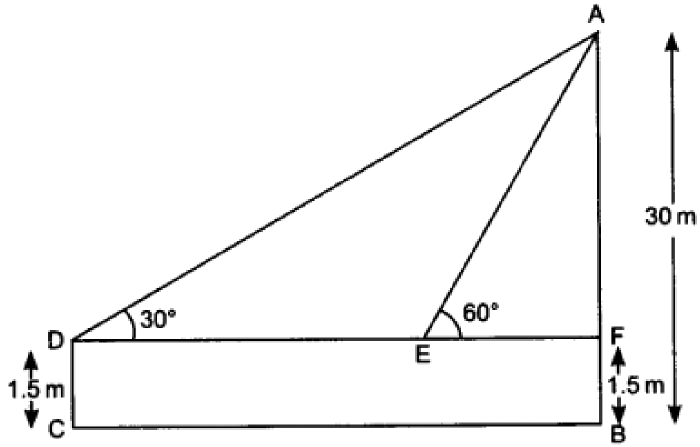
Question 6.
A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Solution:

Let AB = height of the building

Given: $\angle ADF = 30°$, $\angle AEF = 60°$

$$AF = AB - FB \quad [\because CD = BF]$$
$$= 30\text{ m} - 1.5\text{ m} = 28.5\text{ m}$$



In ΔAFE,

$$\frac{AF}{EF} = \tan 60°$$
$$\Rightarrow \frac{28.5}{EF} = \sqrt{3}$$
$$\Rightarrow EF = \frac{28.5}{\sqrt{3}}\text{ m}$$

In ΔAFD,

$$\frac{AF}{DF} = \tan 30°$$
$$\Rightarrow \frac{28.5}{DF} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow DF = 28.5\sqrt{3}\text{ m}$$

The distance walked by the boy towards building

$$DE = DF - EF$$
$$= 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}} = \frac{28.5 \times 3 - 28.5}{\sqrt{3}} = \frac{28.5(3-1)}{\sqrt{3}}$$
$$= \frac{28.5 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{57\sqrt{3}}{3} = 19\sqrt{3}\text{ m}$$

Question 7.

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Solution:

Given: $AB = 20$ m (Height of the building)

Let $AD = h$ m (Height of the tower)

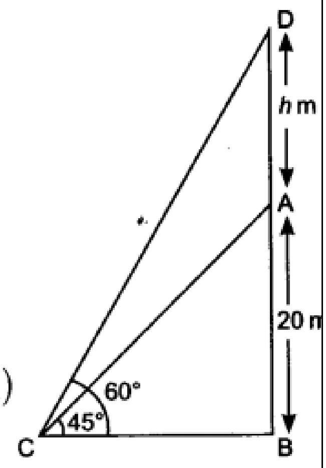
$\angle ACB = 45°$ and $\angle DCB = 60°$

In ΔABC, $\frac{AB}{BC} = \tan 45° \Rightarrow \frac{20}{BC} = 1 \Rightarrow BC = 20$ m

In ΔDBC, $\frac{DB}{BC} = \tan 60° \Rightarrow \frac{20+h}{20} = \sqrt{3}$

$\Rightarrow h + 20 = 20\sqrt{3} \Rightarrow h = 20\sqrt{3} - 20 = 20(\sqrt{3}-1)$

Hence, height of the tower $= 20(\sqrt{3}-1)$ m



Question 8.

A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Solution:

Let the height of the pedestal AB = h m

Given: height of the statue = 1.6 m, ∠ACB = 45° and ∠DCB = 60°

In ΔABC, $\frac{AB}{BC} = \tan 45° \Rightarrow \frac{h}{BC} = 1 \Rightarrow BC = h$

In ΔDBC, $\frac{DB}{BC} = \tan 60°$

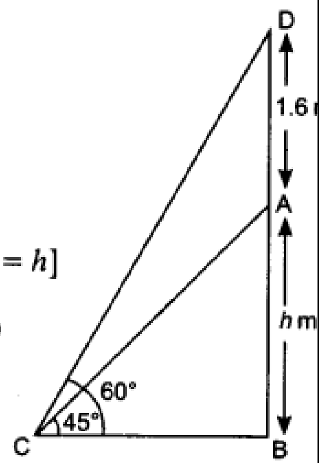
$\Rightarrow \frac{1.6+h}{h} = \sqrt{3}$ $[\because BC = h]$

$\Rightarrow 1.6 + h = \sqrt{3}h \Rightarrow 1.6 = \sqrt{3}h - h \Rightarrow 1.6 = h(\sqrt{3}-1)$

$\Rightarrow \frac{1.6}{\sqrt{3}-1} = h \Rightarrow \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = h$

$\Rightarrow \frac{1.6(\sqrt{3}+1)}{3-1} = h \Rightarrow \frac{1.6(\sqrt{3}+1)}{2} = h \Rightarrow h = 0.8(\sqrt{3}+1)$

Hence, height of the pedestal $= 0.8(\sqrt{3}+1)$m

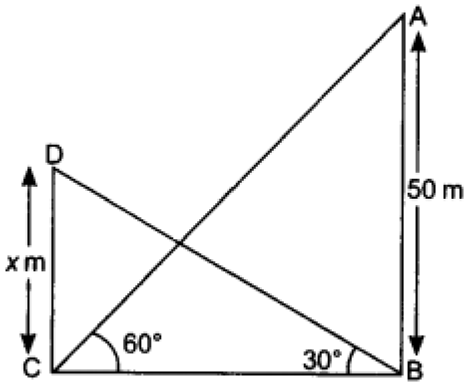


Question 9.

The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Solution:

Given: height of the tower AB = 50 m,
∠ACB = 60°, ∠DBC = 30°
Let the height of the building CD = x m
In ΔABC,



$$\frac{AB}{BC} = \tan 60°$$

$$\Rightarrow \quad \frac{50}{BC} = \sqrt{3} \quad \Rightarrow \quad BC = \frac{50}{\sqrt{3}}$$

In ΔDCB,

$$\Rightarrow \quad \frac{DC}{BC} = \tan 30° \quad \Rightarrow \quad \frac{x}{BC} = \frac{1}{\sqrt{3}}$$

$$x = \frac{1}{\sqrt{3}} \times BC \Rightarrow x = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}} = \frac{50}{3} \text{m}$$

Hence, height of the building $= \frac{50}{3}$ m

Question 10.

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distance of the point from the poles.

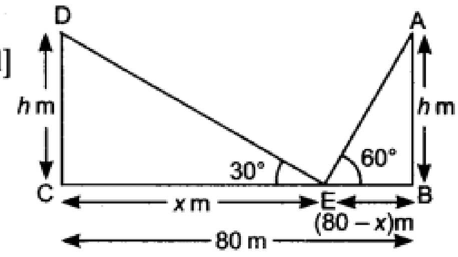
Solution:

Let AB = CD = h m [Height of the poles]

Given: BC = 80 m [Width of the road]

Let CE = x m

$\therefore$ BE = $(80 - x)$ m



In ΔCDE, $\frac{CD}{CE} = \frac{h}{x} = \tan 30°$

$$\frac{h}{x} = \frac{1}{\sqrt{3}} \Rightarrow x = \sqrt{3}h \quad \text{... }(i)$$

In ΔABE, $\frac{AB}{BE} = \tan 60° \Rightarrow \frac{h}{80 - x} = \sqrt{3}$

$$\Rightarrow \quad h = 80\sqrt{3} - \sqrt{3}x \Rightarrow \sqrt{3}x = 80\sqrt{3} - h$$

$$\Rightarrow \quad x = \frac{80\sqrt{3} - h}{\sqrt{3}} \quad \text{... }(ii)$$

From equation (i) and (ii), we get

$$\sqrt{3}h = \frac{80\sqrt{3} - h}{\sqrt{3}} \Rightarrow 3h = 80\sqrt{3} - h \Rightarrow 4h = 80\sqrt{3} \Rightarrow h = 20\sqrt{3}$$

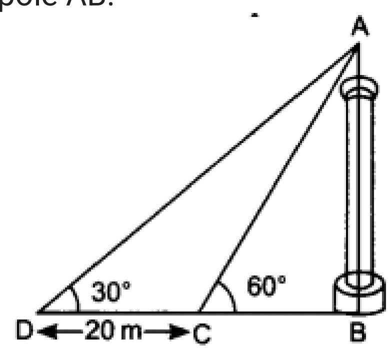
Substituting h in equation (i),

$$x = h\sqrt{3} = 20\sqrt{3} \times \sqrt{3} = 60 \text{ m}$$

Hence, position of the point is at a distance of 60 m from pole CD and 20 m from pole AB.

Question 11.

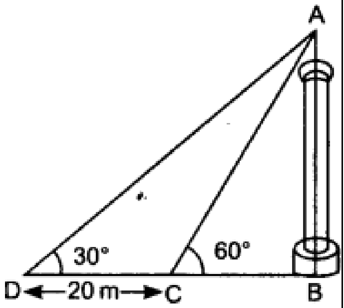
A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see the given figure). Find the height of the tower and the width of the CD and 20 m from pole AB.



Solution:

Let the height of the tower AB = h m and BC be the width of the canal.

Given: $\angle ACB = 60°$ and $\angle ADB = 30°$



In ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow \frac{h}{BC} = \sqrt{3}$

$\Rightarrow BC = \frac{h}{\sqrt{3}}$

In ΔABD, $\frac{AB}{BD} = \tan 30°$

$\Rightarrow \frac{h}{20 + BC} = \frac{1}{\sqrt{3}}$

Putting the value of $BC = \frac{h}{\sqrt{3}}$, we get

$$\frac{h}{20 + \frac{h}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

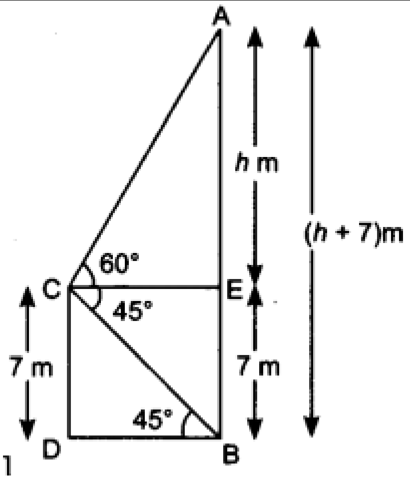
$\Rightarrow 3h = h + 20\sqrt{3} \Rightarrow 2h = 20\sqrt{3} \Rightarrow h = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$

Width of the canal $= \frac{h}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10$ m

Hence, the height of the tower $= 10\sqrt{3}$ m and width of the canal = 10 m.

Question 12.
From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution:
Let height of the tower AB = (h + 7) m
Given: CD = 7 m (height of the building),



$\angle ACE = 60°$, and $\angle ECB = 45°$

$\Rightarrow \quad \angle CBD = 45°$

In ΔCDB, $\dfrac{CD}{DB} = \tan 45° \Rightarrow \dfrac{7}{DB} = 1$

$\Rightarrow \quad DB = 7$ m

In ΔAEC, $\dfrac{AE}{CE} = \tan 60°$

$\Rightarrow \quad \dfrac{h}{7} = \sqrt{3} \qquad [\because DB = CE = 7m]$

$\Rightarrow \quad h = 7\sqrt{3}$ m

Now, $AB = h + 7 = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$m

Hence, height of the tower $= 7(\sqrt{3} + 1)$m

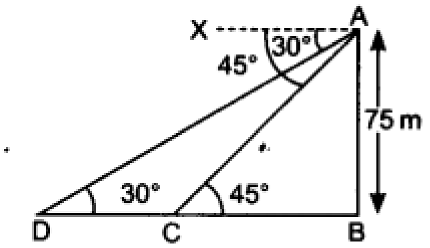
Question 13.

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Solution:

Given: height of the lighthouse = 75 m

Let C and D are the positions of two ships.



We have $\angle XAD = \angle ADB = 30°$

and $\angle XAC = \angle ACB = 45°$

In ΔABC, $\frac{AB}{BC} = \tan 45° \Rightarrow \frac{75}{BC} = 1$

$\Rightarrow$ $BC = 75$ m

In ΔABD, $\frac{AB}{BD} = \tan 30° \Rightarrow \frac{75}{DC+75} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $DC + 75 = 75\sqrt{3}$

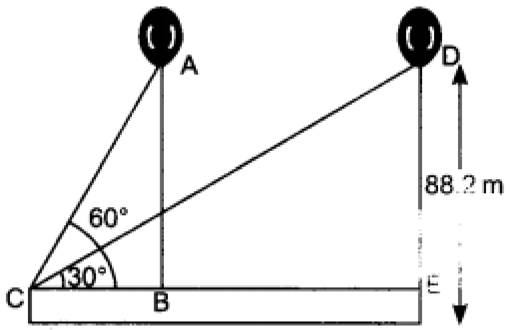
$\Rightarrow$ $DC = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1)$

$= 75(1.73 - 1) = 75 \times 0.73 = 54.75$ m

Hence, the distance between two ships is 54.75 m.

Question 14.

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces to 30° (see figure). Find the distance travelled by the balloon during the interval.



Solution:

Let the first position of the balloon is A and after sometime it will reach to the point D.

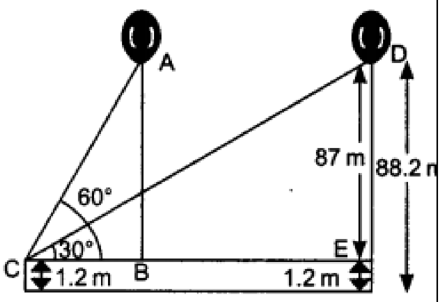
The vertical height ED = AB = (88.2 – 1.2) m = 87 m.

In ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow \frac{87}{BC} = \sqrt{3} \Rightarrow BC = \frac{87}{\sqrt{3}}$

In ΔDEC, $\frac{DE}{CE} = \tan 30°$

$\Rightarrow \frac{87}{CE} = \frac{1}{\sqrt{3}} \Rightarrow CE = 87\sqrt{3}\,m$



Distance travelled by the balloon from A to D is BE.

So, BE = CE – CB

$= 87\sqrt{3} - \frac{87}{\sqrt{3}} = \frac{87(3-1)}{\sqrt{3}} = \frac{87 \times 2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 29 \times 2\sqrt{3} = 58\sqrt{3}\,m$

Question 15.

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Solution:

Let the height of the tower AB = h m

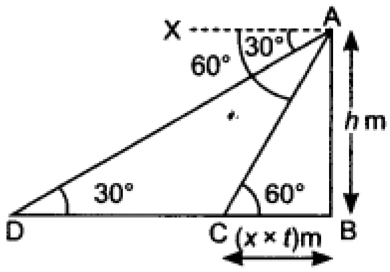
Given: ∠XAD = ∠ADB = 30°

and ∠XAC = ∠ACB = 60°

Let the speed of the car = x m/sec

Distance CD = 6 x x = 6x m

Let the time taken from C to B = t sec.



Distance BC = $x \times t$ m [∵ Distance = Speed × Time]

In ΔABD, $\frac{AB}{DB} = \tan 30° \Rightarrow \frac{h}{DC + CD} = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{6x + tx} = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad h = \frac{(6+t)x}{\sqrt{3}} \quad \text{... (i)}$$

In ΔABC, $\frac{AB}{BC} = \tan 60° \Rightarrow \frac{h}{tx} = \sqrt{3}$

$$\Rightarrow \quad h = tx \times \sqrt{3} \quad \text{... (ii)}$$

From equation (*i*) and (*ii*), we get

$$\frac{(6+t)x}{\sqrt{3}} = tx \times \sqrt{3}$$

$$\Rightarrow \quad (6 + t)x = tx \times 3 \Rightarrow 6x + tx = tx \times 3$$

$$\Rightarrow \quad 6x = 3tx - tx \Rightarrow 6x = 2tx \Rightarrow 6 = 2t \Rightarrow \frac{6}{2} = t \quad \Rightarrow t = 3$$

Hence, time taken from C to B = 3 sec.

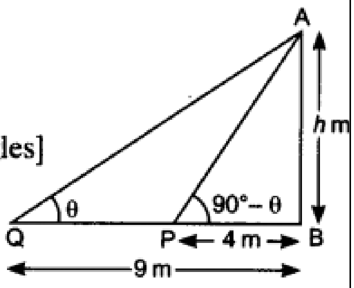
Question 16.

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Solution:

Let the height of the tower AB = h m

We have PB = 4 m, QB = 9 m



Let $\angle AQB = \theta$

Then, $\angle APB = 90° - \theta$ [Both are complementary angles]

In ΔABP, $\frac{AB}{PB} = \tan(90° - \theta)$

$\Rightarrow \frac{h}{4} = \cot\theta$

$\Rightarrow h = 4\cot\theta$ ... (*i*)

In ΔABQ, $\frac{AB}{QB} = \tan\theta$

$\frac{h}{9} = \tan\theta$

$h = 9\tan\theta$ ... (*ii*)

From equation (*i*) and (*ii*), we get

$h \times h = 4\cot\theta \times 9\tan\theta$

$\Rightarrow h^2 = 36\cot\theta \times \tan\theta = 36\frac{1}{\tan\theta} \times \tan\theta$

$\Rightarrow h^2 = 36 \Rightarrow h = 6$ m

Hence, the height of the tower is 6 m.