NCERT Solutions of Chapter 5 – Arithmetic Progressions

Ex 5.1

Question 1.
In which of the following situations, does the list of numbers involved make an arithmetic progression and why?
(i) The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.
(ii) The amount of air present in a cylinder when a vacuum pump removes 14 of the air remaining in the cylinder at a time.
(iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.
(iv) The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8% per annum.

Solution:
(i) Given:
a_1 = ₹ 15
a_2 = ₹ 15 + ₹ 8 = ₹ 23
a_3 = ₹ 23 + ₹ 8 = ₹ 31
List of fares is ₹ 15, ₹ 23, ₹ 31
and $a_2 - a_1$ = ₹ 23 − ₹ 15 = ₹ 8
$a_3 - a_2$ = ₹ 31 − ₹ 23 = ₹ 8
Here, $a_2 - a_1 = a_3 - a_2$
Thus, the list of fares forms an AP.

(ii) Let
$$a_1 = x$$
$$a_2 = x - \frac{1}{4}x = \frac{3}{4}x$$
$$a_3 = \frac{3}{4}x - \frac{1}{4}\left(\frac{3}{4}x\right) = \frac{3}{4}x - \frac{3}{16}x = \frac{9}{16}x$$
The list of numbers is $x, \frac{3}{4}x, \frac{9}{16}x, \ldots$
$$a_2 - a_1 = \frac{3}{4}x - x = -\frac{1}{4}x$$
$$a_3 - a_2 = \frac{9}{16}x - \frac{3}{4}x = -\frac{3x}{16}$$
Here, $a_2 - a_1 \neq a_3 - a_2$
Thus, it is not an AP.

(iii) Given:
a_1 = ₹ 150, a_2 = ₹ 200, a_3 = ₹ 250

$a_2 - a_1$ = ₹ 200 – ₹ 150 = ₹ 50
and $a_3 - a_2$ = ₹ 250 – ₹ 200 = ₹ 50
Here, $a_3 - a_2 = a_2 - a_1$
Thus, the list forms an AP.
(iv) Given: a_1 = ₹ 10000
a_2 = ₹ 10000 + ₹ 10000 x 8100 = ₹ 10000 + ₹ 800 = ₹ 10800
a_3 = ₹ 10800 + ₹ 10800 x 8100 = ₹ 10800 + ₹ 864 = ₹ 11664
$a_2 - a_1$ = ₹ 10800 – ₹ 10000 = ₹ 800
$a_3 - a_2$ = ₹ 11664 – ₹ 10800 = ₹ 864
$a_3 - a_2 \neq a_2 - a_1$
Thus, it is not an AP.

Question 2.
Write first four terms of the AP, when the first term a and the common difference d are given as follows:
(i) a = 10, d = 10
(n) a = -2, d = 0
(iii) a = 4, d = -3
(iv) a = -1, d = 12
(v) a = -1.25, d = -0.25

Solution:
(i) Given: a = 10, d = 10
$a_1 = 10$,
$a_2 = 10 + 10 = 20$
$a_3 = 20 + 10 = 30$
$a_4 = 30 + 10 = 40$
Thus, the first four terms of the AP are 10, 20, 30, 40.

(ii) Given: a = – 2, d = 0
The first four terms of the AP are -2, -2, -2, -2.

(iii) $a_1 = 4$, d = -3
$a_2 = a_1 + d = 4 - 3 = 1$
$a_3 = a_2 + d = 1 - 3 = -2$
$a_4 = a_3 + d = -2 - 3 = -5$
Thus, the first four terms of the AP are 4, 1, -2, -5.

(iv)

$$a_1 = -1, d = \frac{1}{2}$$

$$a_2 = a_1 + d = \frac{-1}{1} + \frac{1}{2} = \frac{-1}{2}$$

$$a_3 = a_2 + d = \frac{-1}{2} + \frac{1}{2} = 0$$

$$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$$

Thus, the first four terms of the AP are $-1, -\frac{1}{2}, 0, \frac{1}{2}$.

(v) a_1 = -1.25, d = -0.25
$a_2 = a_1 + d$ = -1.25 – 0.25 = -1.50
$a_3 = a_2 + d$ = -1.50 – 0.25 = -1.75
$a_4 = a_3 + d$ = -1.75 – 0.25 = -2
Thus, the first four terms of the AP are -1.25, -1.50, -1.75, -2.

Question 3.
For the following APs, write the first term and the common difference:
(i) 3, 1, -1, -3, ……
(ii) -5, -1, 3, 7, ……
(iii) 13 , 53 , 93, 133 , ……..
(iv) 0.6, 1.7, 2.8, 3.9, …….

Solution:
(i) a_1 = 3, a_2 = 1
d = $a_2 - a_1$ = 1 – 3 = -2
where, a_1 = first term and d = common difference
a_1 = 3, d = -2
(ii) a_1 = -5, a_2 = -1
d = $a_2 - a_1$ = -1 – (-5) = -1 + 5 = 4
So, first term a_1 = -5 and common difference d = 4
(iii) a_1 = 13, a_2 = 53
d = 53 – 13 = 43
So, first term a_1 = 13 and common difference d = 43
a_1= 0.6, a_2 = 1.7
d = $a_2 - a_1$ = 1.7 – 0.6 = 1.1
So, first term a_1 = 0.6 and common difference d = 1.1

Question 4.
Which of the following are APs ? If they form an AP, find the common difference d

and write three more terms.
(i) 2, 4, 8, 16, …….
(ii) 2, 52 , 3, 72 , …….
(iii) -1.2, -3.2, -5.2, -7.2, ……
(iv) -10, -6, -2,2, …..
(v) 3, 3 + √2, 3 + 2√2, 3 + 3√2, …..
(vi) 0.2, 0.22, 0.222, 0.2222, ……
(vii) 0, -4, -8, -12, …..
(viii) −12 , −12 , −12 , −12 , …….
(ix) 1, 3, 9, 27, …….
(x) a, 2a, 3a, 4a, …….
(xi) a, a2, a3, a4, …….
(xii) √2, √8, √18, √32, …..
(xiii) √3, √6, √9, √12, …..
(xiv) 12, 32, 52, 72, ……
(xv) 12, 52, 72, 73, ……

Solution:

(i) 2, 4, 8, 16, ……
$a_2 - a_1 = 4 - 2 = 2$
$a_3 - a_2 = 8 - 4 = 4$
$a_2 - a_1 \neq a_3 - a_2$
Thus, the given sequence is not an AP.

(*ii*) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

$$a_2 - a_1 = \frac{5}{2} - \frac{2}{1} = \frac{1}{2}$$

$$a_3 - a_2 = \frac{3}{1} - \frac{5}{2} = \frac{1}{2}$$

$$a_2 - a_1 = a_3 - a_2$$

Thus, the given sequence is an AP.

$$a_1 = 2, d = \frac{1}{2}$$

Next three terms are $a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2} = 4,$

$$a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{9}{2}, a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = 5$$

(*iii*) $-1.2, -3.2, -5.2, -7.2, \ldots$

$$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$$

$$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$$

$$a_3 - a_2 = a_2 - a_1$$

Thus, the given sequence is an AP.

$$a_1 = -1.2, d = -2$$

Next three terms are $a_5 = a_4 + d = -7.2 + (-2) = -9.2,$

$$a_6 = a_5 + d = (-9.2) + (-2) = -11.2$$

$$a_7 = a_6 + d = (-11.2) + (-2) = -13.2$$

(iv) –10, –6, –2, 2, ...

$$a_2 - a_1 = -6 - (-10) = 4$$
$$a_3 - a_2 = -2 - (-6) = 4$$

Thus, the given sequence is an AP.

$$a_1 = -10, d = 4$$

Next three terms are $a_5 = a_4 + d = 2 + 4 = 6, a_6 = a_5 + d = 6 + 4 = 10$

$$a_7 = a_6 + d = 10 + 4 = 14$$

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, ...$

$$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$
$$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$

Thus, the given sequence is an AP.

$$a_1 = 3, d = \sqrt{2}$$

Next three terms are $a_5 = a_4 + d = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$$a_6 = a_5 + d = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$$
$$a_7 = a_6 + d = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

$$a_2 - a_1 = 0.22 - 0.2 = 0.02$$
$$a_3 - a_2 = 0.222 - 0.22 = 0.002$$
$$a_3 - a_2 \neq a_2 - a_1$$

Thus, the given sequence is not an AP.

(vii) 0, – 4, – 8, – 12, ...

$$a_2 - a_1 = -4 - 0 = -4$$
$$a_3 - a_2 = -8 - (-4) = -4$$
$$a_3 - a_2 = a_2 - a_1$$

Thus, the given sequence is an AP.

$$a_1 = 0, d = -4$$

Next three terms are

$$a_5 = a_4 + d = -12 + (-4) = -16$$
$$a_6 = a_5 + d = -16 - 4 = -20$$
$$a_7 = a_6 + d = -20 - 4 = -24$$

(viii) $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \ldots$

$$a_2 - a_1 = \frac{-1}{2} - \left(\frac{-1}{2}\right) = \frac{-1}{2} + \frac{1}{2} = 0$$
$$a_3 - a_2 = \frac{-1}{2} - \left(\frac{-1}{2}\right) = 0$$
$$a_3 - a_2 = a_2 - a_1$$

Thus, the given sequence is an AP.

$$a_1 = \frac{-1}{2}, d = 0$$

Next three terms are $a_5 = a_4 + d = \frac{-1}{2}, a_6 = a_5 + d = \frac{-1}{2}, a_7 = a_6 + d = \frac{-1}{2}$

(ix) 1, 3, 9, 27, ...

$$a_2 - a_1 = 3 - 1 = 2$$
$$a_3 - a_2 = 9 - 3 = 6$$
$$a_3 - a_2 \neq a_2 - a_1$$

Thus, the given sequence is not an AP.

(x) $a, 2a, 3a, 4a, \ldots$

$$a_2 - a_1 = 2a - a = a$$
$$a_3 - a_2 = 3a - 2a = a$$
$$a_2 - a_1 = a_3 - a_2$$

Thus, the given sequence is an AP.

$$a_1 = a, d = a$$

Next three terms are $a_5 = a_4 + d = 4a + a = 5a$

$$a_6 = a_5 + d = 5a + a = 6a$$
$$a_7 = a_6 + d = 6a + a = 7a$$

(*xi*) $a, a^2, a^3, a^4, \ldots$

$$a_2 - a_1 = a^2 - a = a(a-1)$$
$$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$$
$$a_3 - a_2 \neq a_2 - a_1$$

Thus, the given sequence is not an AP.

(*xii*) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots$

$$a_2 - a_1 = \sqrt{8} - \sqrt{2}$$
$$= 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$
$$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$
$$a_3 - a_2 = a_2 - a_1$$

Thus, the given sequence is an AP.

Next three terms are

$$a_5 = a_4 + d = 32 + \sqrt{2}$$
$$= 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$$
$$a_6 = a_5 + d = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$$
$$a_7 = a_6 + d = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$$

(*xiii*) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$

$$a_2 - a_1 = \sqrt{6} - \sqrt{3}$$
$$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$$
$$a_3 - a_2 \neq a_2 - a_1$$

Thus, the given sequence is not an AP.

(xiv) $1^2, 3^2, 5^2, 7^2, ...$

$$a_2 - a_1 = 9 - 1 = 8$$
$$a_3 - a_2 = 25 - 9 = 16$$
$$a_3 - a_2 \neq a_2 - a_1$$

Thus, the given sequence is not an AP.

(xv) $1^2, 5^2, 7^2, 73, ...$

$$a_2 - a_1 = 25 - 1 = 24$$
$$a_3 - a_2 = 49 - 25 = 24$$

Thus, the given sequence is an AP.

$$a_1 = 1, d = 24$$

Next three terms are

$$a_5 = a_4 + d = 73 + 24 = 97$$
$$a_6 = a_5 + d = 97 + 24 = 121$$
$$a_7 = a_6 + d = 121 + 24 = 145$$

Ex 5.2 Class 10 Maths Solutions Question 1.

Fill in the blanks in the following table, given that a is the first term, d the common difference and an the nth term of the AP:

	a	d	n	a_n
(i)	7	3	8	...
(ii)	–18	...	10	0
(iii)	...	–3	18	–5
(iv)	–18.9	2.5	...	3.6
(v)	3.5	0	105	...

Solution:

(i) $a = 7, d = 3, n = 8$

$$a_n = a + (n-1)d \Rightarrow a_8 = 7 + (8-1)\,3 = 7 + 21 = 28$$

(ii) $a = -18, n = 10, a_n = 0$

$$a_n = a + (n-1)d \Rightarrow 0 = -18 + (10-1)\,d$$

$$\Rightarrow \quad 0 = -18 + 9d \Rightarrow 18 = 9d \Rightarrow \frac{18}{9} = d$$

$$d = 2$$

(iii) $d = -3, n = 18, a_n = -5$

$$a_n = a + (n-1)\,d \Rightarrow -5 = a + (18-1)\,(-3) \Rightarrow -5 = a - 51$$

$$\Rightarrow \quad -5 + 51 = a \Rightarrow a = 46$$

(iv) $a = -18.9, d = 2.5, a_n = 3.6$

$$a_n = a + (n-1)\,d \Rightarrow 3.6 = -18.9 + (n-1)\,2.5$$

$$\Rightarrow \quad 3.6 + 18.9 = (n-1)\,2.5 \Rightarrow \frac{22.5}{2.5} = n - 1 \Rightarrow 9 = n - 1 \Rightarrow n = 9 + 1 = 10$$

(v) $a = 3.5, d = 0, n = 105$

$$a_n = a + (n-1)\,d = 3.5 + (105-1)\,0 = 3.5 + 0 = 3.5$$

Class 10 Maths Chapter 5 Exercise 5.2 Question 2.

Choose the correct choice in the following and justify:

(i) 30th term of the AP: 10, 7, 4, …, is

(A) 97

(B) 77

(C) -77

(D) -87

(ii) 11th term of the AP: -3, $-\frac{1}{2}$, 2, …, is

(A) 28

(B) 22

(C) -38

(D) -48

Solution:

(i) 10, 7, 4, …,

a = 10, d = 7 − 10 = -3, n = 30

$a_n = a + (n - 1)d$

$\Rightarrow a_{30} = a + (30 - 1)\,d = a + 29\,d = 10 + 29\,(-3) = 10 - 87 = -77$

Hence, correct option is **(C).**

(*ii*) $-3, -\frac{1}{2}, 2, \ldots,$

$$a = -3, n = 11$$

$$d = -\frac{1}{2} - (-3) = -\frac{1}{2} + \frac{3}{1} = \frac{5}{2}$$

$$a_n = a + (n-1)d \Rightarrow a_{11} = a + (11-1)d$$

$$\Rightarrow \quad a_{11} = a + 10d = -3 + 10 \times \frac{5}{2} = -3 + 25 = 22$$

Hence, correct option is (B).

Class 10 Maths Chapter 5 Exercise 5.2 Question 3.

In the following APs, find the missing terms in the boxes:

(*i*) $2, \square, 26$

(*ii*) $\square, 13, \square, 3$

(*iii*) $5, \square, \square, 9\frac{1}{2}$

(*iv*) $-4, \square, \square, \square, \square, 6$

(*v*) $\square, 38, \square, \square, \square, -22$

Solution:

(i)

$$a = 2, a_3 = 26$$

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad a + 2d = 26 \Rightarrow 2 + 2d = 26 \Rightarrow 2d = 26 - 2$$

$$\Rightarrow \quad 2d = 24 \Rightarrow d = \frac{24}{2} = 12$$

$$a_2 = a + (2-1)d = a + d = 2 + 12 = 14$$

(ii)

$$a_2 = 13 \Rightarrow a_2 = a + (2-1)d \Rightarrow a + d = 13 \quad \text{... }(i)$$

$$a_4 = 3 \Rightarrow a_4 = a + (4-1)d \Rightarrow a + 3d = 3 \quad \text{... }(ii)$$

Subtracting (*i*) and (*ii*), we get

$$a + d - a - 3d = 13 - 3 \Rightarrow -2d = 10 \Rightarrow d = \frac{10}{-2} = -5$$

From (*i*),

$$a + d = 13 \Rightarrow a + (-5) = 13 \Rightarrow a = 13 + 5 \Rightarrow a = 18$$

$$a_3 = a + 2d = 18 + 2 \times (-5) = 18 - 10 = 8$$

(iii) **Given:**

$$a = 5, a_4 = 9\frac{1}{2}$$

$$\therefore \quad a = 5 \text{ and } a + 3d = \frac{19}{2} \Rightarrow 5 + 3d = \frac{19}{2}$$

$$\Rightarrow \quad 3d = \frac{19}{2} - \frac{5}{1} \Rightarrow 3d = \frac{19-10}{2} \Rightarrow 3d = \frac{9}{2} \Rightarrow d = \frac{9}{2 \times 3} = \frac{3}{2}$$

$$a_2 = a + (2-1)d \Rightarrow a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$$

$$a_3 = a_2 + d = \frac{13}{2} + \frac{3}{2} = \frac{16}{2} = 8$$

(iv) Given:

$$a = -4, a_6 = 6$$

$$a_6 = a + (6-1)d \Rightarrow a + 5d = 6 \Rightarrow -4 + 5d = 6$$

$$\Rightarrow \quad 5d = 6 + 4 \Rightarrow 5d = 10 \Rightarrow d = \frac{10}{5} = 2$$

$$a_2 = a + d = -4 + 2 = -2$$

$$a_3 = a_2 + d = -2 + 2 = 0$$

$$a_4 = a_3 + d = 0 + 2 = 2$$

$$a_5 = a_4 + d = 2 + 2 = 4$$

(*v*) **Given:** $a_2 = 38, \quad a_6 = -22$

$$a_2 = a + d \Rightarrow a + d = 38 \quad ...(i)$$

$$a_6 = a + (6-1)d \Rightarrow a + 5d = -22 \quad ...(ii)$$

Subtracting (*i*) and (*ii*), we get

$$\therefore \quad a + 5d - a - d = -22 - 38 \Rightarrow 4d = -60 \Rightarrow d = \frac{-60}{4} = -15$$

$$a + d = 38 \Rightarrow a + (-15) = 38 \Rightarrow a = 38 + 15 = 53$$

$$a_3 = a + (3-1)d = a + 2d = 53 + 2 \times (-15) = 53 - 30 = 23$$

$$a_4 = a_3 + d = 23 - 15 = 8$$

$$a_5 = a_4 + d = 8 - 15 = -7$$

Class 10 Maths Chapter 5 Exercise 5.2 Question 4.

Which term of the AP: 3, 8, 13, 18, …, is 78?

Solution:

Given: 3, 8, 13, 18, ………,

a = 3, d = 8 − 3 = 5

Let nth term is 78

$a_n = 78$

a + (n − 1) d = 78

⇒ 3 + (n − 1) 5 = 78

⇒ (n − 1) 5 = 78 − 3

⇒ (n − 1) 5 = 75

⇒ n − 1 = 15

⇒ n = 15 + 1

⇒ n = 16

Hence, a_{16} = 78

Class 10 Maths Chapter 5 Exercise 5.2 Question 5.

Find the number of terms in each of the following APs:

(i) 7, 13, 19, …, 205

(ii) 18, 1512, 13, …, -47

Solution:

(i) Let there are n terms in the AP, 7, 13, 19, ..., 205

Here, $a = 7, d = 13 - 7 = 6, a_n = 205$

We have, $a_n = a + (n-1)d$

$\Rightarrow 205 = 7 + (n-1)6 \Rightarrow 205 - 7 = (n-1)6$

$\Rightarrow \frac{198}{6} = n - 1 \Rightarrow 33 + 1 = n \Rightarrow n = 34$

(ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

$a = 18, d = \frac{31}{2} - \frac{18}{1} = \frac{-5}{2}, a_n = -47$

$a_n = a + (n-1)d \Rightarrow -47 = 18 + (n-1)\left(\frac{-5}{2}\right)$

$\Rightarrow -47 - 18 = (n-1)\left(\frac{-5}{2}\right) \Rightarrow -65 \times \left(\frac{2}{-5}\right) = n - 1$

$\Rightarrow 26 = n - 1 \Rightarrow 26 + 1 = n \Rightarrow 27 = n$

Class 10 Ex 5.2 Solutions Question 6.

Check, whether -150 is a term of the AP: 11, 8, 5, 2, ….

Solution:

11, 8, 5, 2, …….

Here, a = 11, d = 8 – 11= -3, a_n = -150

a + (n – 1) d = an

⇒ 11 + (n – 1) (- 3) = -150

⇒ (n – 1) (- 3) = -150 – 11

⇒ -3 (n – 1) = -161

⇒ n – 1 = −161−3

⇒ n = 1613 + 1 = 1643 = 5343

Which is not an integral number.

Hence, -150 is not a term of the AP.

Maths NCERT Solutions Class 10 Arithmetic Progression Exercise 5.2 Question 7.

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

a_{11} = 38 and a_{16} = 73

⇒ a_{11} = a + (11 – 1) d ⇒ a + 10d = 38 ….. (i)

⇒ a_{16} = a + (16 – 1)d ⇒ a + 15d = 73 …(ii)

Subtracting eqn. (i) from (ii), we get

a + 15d – a – 10d = 73 – 38

⇒ 5d = 35

⇒ d = 1

From (i), a + 10 x 7 = 38

$\Rightarrow$ a = 38 – 70 = – 32
a_{31} = a + (31 – 1) d = a + 30d = – 32 + 30 x 7 = – 32 + 210 = 178

Class 10 Maths Chapter 5 Exercise 5.2 Question 8.
An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.
Solution:
Given:
a_{50} = 106
a_{50} = a + (50 – 1) d
$\Rightarrow$ a + 49d = 106 ...(i)
and a_3 = 12 $\Rightarrow$ a_3 = a + (3 – 1)d $\Rightarrow$ a + 2d = 12 ...(ii)
Subtracting eqn. (ii) from (i), we get
a + 49d – a – 2d = 106 – 12
$\Rightarrow$ 47d = 94
$\Rightarrow$ d = 9447 = 2
a + 2d = 12
$\Rightarrow$ a + 2 x 2 = 12
$\Rightarrow$ a + 4 = 12
$\Rightarrow$ a = 12 – 4 = 8
a_{29} = a + (29 – 1) d = a + 28d = 8 + 28 x 2 = 8 + 56 = 64

Ex 5.2 Class 10 NCERT Solutions Question 9.
If the 3rd and the 9th term of an AP are 4 and -8 respectively, which term of this AP is zero?
Solution:
Given: a_3 = 4 and a_9 = – 8
$\Rightarrow$ a_3 = a + (3 – 1)d $\Rightarrow$ a + 2d = 4 ...(i)
a_9 = a + (9 – 1) d $\Rightarrow$ a + 8d = -8 ....(ii)
Subtracting eqn. (i) from (ii), we get
a + 8d – a – 2d = -8 – 4
$\Rightarrow$ 6d = -12.
$\Rightarrow$ d = -2
Now,
a + 2d = 4
$\Rightarrow$ a + 2(-2) = 4
$\Rightarrow$ a – 4 = 4
$\Rightarrow$ a = 4 + 4 = 8
Let a_n = 0
$\Rightarrow$ a + (n – 1) d = 0
$\Rightarrow$ 8 + (n – 1) (- 2) = 0
$\Rightarrow$ 8 = 2 (n – 1)
$\Rightarrow$ n – 1 = 4

⇒ n = 4 + 1 = 5
Hence, 5th term is zero.

Exercise 5.2 Class 10 NCERT Solutions Question 10.
The 17th term of an AP exceeds its 10th term by 7. Find the common difference.
Solution:
Given: $a_{17} - a_{10} = 7$
⇒ [a + (17 − 1) d] − [a + (10 − 1) d] = 7
⇒ (a + 16d) − (a + 9d) = 7
⇒ 7d = 7
⇒ d = 1

Class 10 Maths Chapter 5 Exercise 5.2 Question 11.
Which term of the AP: 3, 15, 27, 39, … will be 132 more than its 54th term?
Solution:
3, 15, 27, 39, …..
Here, a = 3, d = 15 − 3 = 12
Let $a_n = 132 + a_{54}$
⇒ $a_n - a_{54} = 132$
⇒ [a + (n − 1) d] − [a + (54 − 1) d] = 132
⇒ a + nd − d − a − 53d = 132
⇒ 12n − 54d = 132
⇒ 12n − 54 x 12 = 132
⇒ (n − 54)12 = 132
⇒ n − 54 = 11
⇒ n = 11 + 54 = 65

Class 10 Maths Chapter 5 Exercise 5.2 Question 12.
Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
Solution:
Let a and A be the first term of two APs and d be the common difference.
Given:
$a_{100} - A_{100} = 100$
⇒ a + 99d − A − 99d = 100
⇒ a − A = 100
⇒ $a_{1000} - A_{1000}$ = a + 999d − A − 999d
⇒ a − A = 100
⇒ $a_{1000} - A_{1000} = 100$

NCERT Solutions Class 10 Maths Ch 5 Ex 5.2 Question 13.
How many three-digit numbers are divisible by 7?
Solution:
The three-digit numbers which are divisible by 7 are 105, 112, 119, ………., 994
Here, $a = 105$, $d = 112 – 105 = 7$, $a_n = 994$
$a + (n – 1) d = 994$
$\Rightarrow 105 + (n – 1) 7 = 994$
$\Rightarrow (n – 1) 7 = 994 – 105$
$\Rightarrow 7 (n – 1) = 889$
$\Rightarrow n – 1 = 127$
$\Rightarrow n = 127 + 1 = 128$

CBSE Class 10 Maths Ex 5.2 Solutions Question 14.
How many multiples of 4 lie between 10 and 250?
Solution:
The multiples of 4 between 10 and 250 be 12, 16, 20, 24,…., 248
Here, $a = 12$, $d = 16 – 12 = 4$, $a_n = 248$
$a_n = a + (n – 1) d$
$\Rightarrow 248 = 12 + (n – 1) 4$
$\Rightarrow 248 – 12 = (n – 1) 4$
$\Rightarrow 236 = (n – 1) 4$
$\Rightarrow 59 = n – 1$
$\Rightarrow n = 59 + 1 = 60$

NCERT Solutions For Class 10 Maths Ch 5 Ex 5.2 Question 15.
For what value of n, the nth term of two APs: 63, 65, 61,… and 3, 10, 17,… are equal?
Solution:
First AP
63, 65, 67,…
Here, $a = 63$, $d = 65 – 63 = 2$
$a_n = a + (n – 1) d = 63 + (n – 1) 2 = 63 + 2n – 2 = 61 + 2n$
Second AP
3, 10, 17, …
Here, $a = 3$, $d = 10 – 3 = 7$
$a_n = a + (n – 1) d = 3 + (n – 1)7 = 3 + 7n – 7 = 7n – 4$
Now, $a_n = a_n$
$\Rightarrow 61 + 2n = 7n – 4$
$\Rightarrow 61 + 4 = 7n – 2n$
$\Rightarrow 65 = 5n$
$\Rightarrow n = 13$

Class 10 Maths Chapter 5.2 Question 16. Determine the AP whose 3rd term is 16 and 7th term exceeds the 5th term by 12. **Solution:** Given: $a_3 = 16$ ⇒ a + (3 – 1)d = 16 ⇒ a + 2d = 16 and $a_7 - a_5 = 12$ ⇒ [a + (7 – 1)d] – [a + (5 – 1)d] = 12 ⇒ a + 6d – a – 4d = 12 ⇒ 2d = 12 ⇒ d = 6 Since a + 2d = 16 ⇒ a + 2(6) = 16 ⇒ a + 12 = 16 ⇒ a = 16 – 12 = 4 $a_1 = a = 4$ $a_2 = a_1 + d = a + d = 4 + 6 = 10$ $a_3 = a_2 + d = 10 + 6 = 16$ $a_4 = a_3 + d = 16 + 6 = 22$ Thus, the required AP is a_1, a_2, a_3, a_4,…, i.e. 4, 10, 16, 22
Class 10 Maths Chapter 5 Exercise 5.2 Question 17. Find the 20th term from the last term of the AP: 3, 8, 13, …, 253. **Solution:** Given: AP is 3, 8, 13,…….. ,253 On reversing the given A.P., we have 253, 248, 243 ,………, 13, 8, 3. Here, a = 253, d = 248 – 253 = -5 a_{20} = a + (20 – 1)d = a + 19d = 253 + 19 (-5) = 253 – 95 = 158
Exercise 5.2 Class 10 Solutions Question 18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Solution:

Given: $a_4 + a_8 = 24$ and $a_6 + a_{10} = 44$

$\Rightarrow a + (4-1)d + a + (8-1)d = 24$

$\Rightarrow a + 3d + a + 7d = 24$ and $a + (6-1)d + a + (10-1)d = 44$

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 10d = 24$ ...(i) and $2a + 14d = 44$ ...(ii)

Subtracting eqn (i) from (ii), we get

$$2a + 14d - 2a - 10d = 44 - 24 \Rightarrow 4d = 20 \Rightarrow d = \frac{20}{4} = 5$$

$$2a + 10d = 24 \Rightarrow 2a + 10 \times 5 = 24$$

$$2a = 24 - 50$$

$$2a = -26$$

$$a = \frac{-26}{2} = -13$$

$$a_1 = a = -13$$

$$a_2 = a_1 + d = -13 + 5 = -8$$

$$a_3 = a_2 + d = -8 + 5 = -3$$

Hence, the first three terms are, –13, –8, –3

Solution Of Ex 5.2 Class 10 Question 19.

Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000 ?

Solution:

a = ₹ 5000, d = ₹ 200

Let a_n = ₹ 7000

We have, a + (n – 1) d = 7000

⇒ 5000 + (n – 1) 200 = 7000

⇒ (n – 1) 200 = 7000 – 5000

⇒ (n – 1) 200 = 2000

⇒ (n- 1) = 10

⇒ n = 11

⇒ 1995 + 11 = 2006

Hence, in 2006 Subba Rao's income will reach ₹ 7000.

NCERT Solutions For Class 10 Maths 5.2 Question 20.

Ramkali saved ₹ 5 in the first week of a year and then increased her weekly saving by ₹ 1.75. If in the nth week, her weekly saving become ₹ 20.75, find n.

Solution:

Given: a = ₹ 5, d = ₹ 1.75

a_n = ₹ 20.75

a + (n – 1) d – 20.75

⇒ 5 + (n – 1) 1.75 = 20.75
⇒ (n – 1) x 1.75 = 20.75 – 5
⇒ (n – 1) 1.75 = 15.75
⇒ n – 1 = 9
⇒ n = 9 + 1
⇒ n = 10
Hence, in 10th week Ramkali's saving will be ₹ 20.75.

Ex 5.3

Question 1.
Find the sum of the following APs:
(i) 2, 7, 12,…… to 10 terms.
(ii) -37, -33, -29, …… to 12 terms.
(iii) 0.6, 1.7, 2.8, ……, to 100 terms.
(iv) 115, 112, 110, …….., to 11 terms.
Solution:

(*i*) 2, 7, 12, ... to 10 terms.

Here, $a = 2, d = 7 - 2 = 5, n = 10, S_{10} = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5] = 5[4 + 45] = 5 \times 49 = 245$$

(*ii*) –37, –33, –29, ... to 12 terms.

Here, $a = -37, d = -33 - (-37) = 4, n = 12, S_{12} = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times (-37) + (12-1)4]$$

$$= 6[-74 + 44] = 6 \times (-30) = -180$$

(*iii*) 0.6, 1.7, 2.8, ... to 100 terms.

Here, $a = 0.6, d = 1.7 - 0.6 = 1.1, n = 100, S_{100} = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{100} = \frac{100}{2}[2 \times 0.6 + (100-1)1.1]$$

$$= 50[1.2 + 108.9] = 50 \times 110.1 = 5505$$

(*iv*) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$, to 11 terms

Here, $a = \frac{1}{15}, n = 11, S_{11} = ?$

$$d = \frac{1}{12} - \frac{1}{15} = \frac{15-12}{180} = \frac{3}{180} = \frac{1}{60}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{11} = \frac{11}{2}\left[2 \times \frac{1}{15} + (11-1)\frac{1}{60}\right] = \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right] = \frac{11}{2}\left[\frac{8+10}{60}\right] = \frac{11}{2} \times \frac{18}{60} = \frac{33}{20}$$

Question 2.

Find the sums given below:

(i) 7 + 1012 + 14 + … + 84

(ii) 34 + 32 + 30 + … + 10

(iii) -5 + (-8) + (-11) + ….. + (-230)

Solution:

(i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots + 84$

Here, $a = 7, d = \frac{21}{2} - \frac{7}{1} = \frac{7}{2}, a_n = 84$

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 84 = 7 + (n-1)\frac{7}{2} \Rightarrow 84 - 7 = (n-1)\frac{7}{2}$$

$$\Rightarrow \quad 77 \times \frac{2}{7} = n - 1 \Rightarrow 22 + 1 = n \Rightarrow n = 23$$

$$S_n = \frac{n}{2}[a + l] \qquad [\because a_n = l]$$

$$S_{23} = \frac{23}{2}[7 + 84] = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046.5$$

(ii) $34 + 32 + 30 + \ldots\ldots + 10$

Here, $a = 34, d = 32 - 34 = -2, a_n = 10$

$$a_n = a + (n-1)d \Rightarrow 10 = 34 + (n-1)(-2)$$

$$\Rightarrow \quad 10 - 34 = (n-1)(-2) \Rightarrow \frac{-24}{-2} = n - 1 \Rightarrow n - 1 = 12$$

$$n = 12 + 1 = 13$$

$$S_{13} = \frac{13}{2}[34 + 10] = \frac{13}{2} \times 44 = 13 \times 22 = 286$$

(iii) $-5 + (-8) + (-11) + \ldots\ldots + (-230)$

Here, $a = -5, a_n = -230,$

$$d = -8 - (-5) = -3$$

$$a_n = a + (n-1)d \Rightarrow -230 = -5 + (n-1)(-3)$$

$$\Rightarrow \quad \frac{-225}{-3} = n - 1 \Rightarrow 75 + 1 = n \Rightarrow n = 76$$

$$S_{76} = \frac{76}{2}[-5 + (-230)] = 38 \times (-235) = -8930$$

Question 3.

In an AP:

(i) given a = 5, d = 3, a_n = 50, find n and S_n.
(ii) given a = 7, a_{13} = 35, find d and S_{13}.
(iii) given a_{12} = 37, d = 3, find a and S_{12}.
(iv) given a_3 = -15, S_{10} = 125, find d and a_{10}.
(v) given d = 5, S_9 = 75, find a and a_9.
(vi) given a = 2, d = 8, S_n = 90, find n and a_n.
(vii) given a = 8, a_n = 62, S_n = 210, find n and d.
(viii) given a_n = 4, d = 2, S_n = -14, find n and a.
(ix) given a = 3, n = 8, S = 192, find d.
(x) given l = 28, S = 144, and there are total 9 terms. Find a.

Solution:

(i) $a = 5, d = 3, a_n = 50$

$a_n = 50 \Rightarrow a + (n-1)d = 50 \Rightarrow 5 + (n-1)3 = 50$

$\Rightarrow (n-1)3 = 50 - 5 \Rightarrow (n-1)3 = 45 \Rightarrow n - 1 = \frac{45}{3}$

$\Rightarrow n - 1 = 15 \quad \Rightarrow \quad n = 16$

$S_{16} = \frac{16}{2}[5 + 50] = 8 \times 55 = 440$

(ii) $a = 7, a_{13} = 35$

$a_{13} = 35 \Rightarrow a + 12d = 35 \Rightarrow 7 + 12d = 35$

$\Rightarrow 12d = 35 - 7 \Rightarrow 12d = 28 \Rightarrow d = \frac{28}{12} = \frac{7}{3}$

$S_{13} = \frac{13}{2}[a + a_{13}] = \frac{13}{2}[7 + 35] = \frac{13}{2} \times 42$

$= 13 \times 21 = 273$

(iii) $a_{12} = 37, d = 3$

$a_{12} = 37 \Rightarrow a + 11d = 37 \Rightarrow a + 11 \times 3 = 37$

$\Rightarrow a = 37 - 33 = 4$

$S_{12} = \frac{12}{2}[a + a_{12}] = 6[4 + 37] = 6 \times 41 = 246$

(iv) $a_3 = 15, S_{10} = 125$

$a_3 = 15 \Rightarrow a + 2d = 15 \Rightarrow a = 15 - 2d$

We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + (n-1)d]$

$= 5[2(15 - 2d) + (10 - 1) \times d] = 5[30 - 4d + 9d]$

$125 = 5(30 + 5d)$

$\frac{125}{5} = 30 + 5d \Rightarrow 25 - 30 = 5d \Rightarrow -5 = 5d$

$$\Rightarrow \quad \frac{-5}{5} = d \Rightarrow d = -1$$

$$a_{10} = a + 9d = 15 - 2d + 9d = 15 + 7d$$

$$= 15 + 7 \times (-1) = 15 - 7 = 8$$

(*v*) $d = 5, S_9 = 75$

$$S_9 = \frac{9}{2}[2a + (9-1)d] \Rightarrow 75 = \frac{9}{2}[2a + 8d]$$

$$\Rightarrow \quad 75 \times \frac{2}{9} = 2a + 8 \times 5 \Rightarrow \frac{50}{3} = 2a + 40$$

$$\frac{50}{3} - \frac{40}{1} = 2a$$

$$\frac{50 - 120}{3} = 2a \Rightarrow \frac{-70}{3 \times 2} = a \Rightarrow a = \frac{-35}{3}$$

$$a_9 = a + 8d = \frac{-35}{3} + 8 \times 5 = \frac{-35}{3} + \frac{40}{1} = \frac{-35 + 120}{3} = \frac{85}{3}$$

(*vi*) $a = 2, d = 8, S_n = 90$

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 90 = \frac{n}{2}[2 \times 2 + (n-1)8]$$

$$\Rightarrow \quad 90 = \frac{n}{2}[4 + 8n - 8] \Rightarrow 90 \times 2 = n[8n - 4]$$

$$\Rightarrow \quad 180 = 8n^2 - 4n \Rightarrow 8n^2 - 4n - 180 = 0$$

$$\Rightarrow \quad 2n^2 - n - 45 = 0 \Rightarrow 2n^2 - 10n + 9n - 45 = 0$$

$$\Rightarrow \quad 2n(n-5) + 9(n-5) = 0 \Rightarrow (2n+9)(n-5) = 0$$

$$\Rightarrow \quad 2n + 9 = 0 \text{ or } n - 5 = 0 \Rightarrow 2n = -9 \text{ or } n = 5$$

$$\Rightarrow \quad n = \frac{-9}{2} \text{(rejected) or } n = 5$$

$$a_n = a + (n-1)d$$

$$a_5 = 2 + (5-1)8 = 2 + 32 = 34$$

(*vii*) $a = 8, a_n = 62, S_n = 210$

$$S_n = \frac{n}{2}[a + a_n] \Rightarrow 210 = \frac{n}{2}[8 + 62] \Rightarrow 210 \times 2 = 70n$$

$$\Rightarrow \quad \frac{210 \times 2}{70} = n \Rightarrow n = 6$$

$$a_6 = 62 \Rightarrow a + 5d = 62$$

$$\Rightarrow \quad 8 + 5d = 62 \Rightarrow 5d = 62 - 8 \Rightarrow 5d = 54 \Rightarrow d = \frac{54}{5} = 10.8$$

(*viii*) $a_n = 4, d = 2, S_n = -14$

$$S_n = \frac{n}{2}[a + a_n] \Rightarrow -14 = \frac{n}{2}[a + 4]$$

$$\Rightarrow \quad -28 = na + 4n \Rightarrow \frac{-28 - 4n}{n} = a$$

$$a_n = 4 \Rightarrow a + (n - 1)d = 4$$

$$\Rightarrow \quad \frac{-4n - 28}{n} + (n - 1)2 = 4 \Rightarrow -4n - 28 + 2n^2 - 2n = 4n$$

$$\Rightarrow \quad 2n^2 - 10n - 28 = 0 \Rightarrow n^2 - 5n - 14 = 0$$

$$\Rightarrow \quad n^2 - 7n + 2n - 14 = 0 \Rightarrow n(n - 7) + 2(n - 7) = 0$$

$$\Rightarrow \quad (n + 2)(n - 7) = 0$$

$$n + 2 = 0 \text{ or } n - 7 = 0$$

$$n = -2 (n = -2 \text{ is rejected}) \text{ or } n = 7$$

So, $n = 7$

$$\therefore \quad a = \frac{-28 - 4n}{n} = \frac{-28 - 4 \times 7}{7} = \frac{-28 - 28}{7} = \frac{-56}{7} = -8$$

(*ix*) $a = 3, n = 8, S = 192$

We have, $S_n = \frac{n}{2}[2a + (n - 1)d] = 192$

$$\Rightarrow \quad \frac{8}{2}[2 \times 3 + (8 - 1)d] = 192 \Rightarrow 4[6 + 7d] = 192 \Rightarrow 6 + 7d = \frac{192}{4}$$

$$\Rightarrow \quad 6 + 7d = 48 \Rightarrow 7d = 48 - 6 \Rightarrow 7d = 42 \Rightarrow d = \frac{42}{7} = 6$$

(*x*) $l = 28, S = 144, n = 9$

We have, $S_n = \frac{n}{2}[a + l]$

$$144 = \frac{9}{2}[a + 28] \Rightarrow 144 \times \frac{2}{9} = a + 28$$

$$\Rightarrow \quad 32 = a + 28 \Rightarrow a = 32 - 28 = 4$$

Question 4.

How many terms of AP: 9, 17, 25, … must be taken to give a sum of 636?

Solution:

Given: $a = 9, d = 17 - 9 = 8, \quad S_n = 636$

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 636 = \frac{n}{2}[2 \times 9 + (n-1)8]$$

$$\Rightarrow \quad 636 \times 2 = n[18 + 8n - 8] \Rightarrow 636 \times 2 = n(10 + 8n)$$

$$\Rightarrow \quad 636 \times 2 = 2n(5 + 4n) \Rightarrow \frac{636 \times 2}{2} = 5n + 4n^2$$

$$\Rightarrow \quad 4n^2 + 5n - 636 = 0 \Rightarrow 4n^2 + 53n - 48n - 636 = 0$$

$$\Rightarrow \quad n(4n + 53) - 12(4n + 53) = 0 \Rightarrow (4n + 53)(n - 12) = 0$$

$$\Rightarrow \quad 4n + 53 = 0 \quad \text{or} \quad n - 12 = 0$$

$$\Rightarrow \quad n = \frac{-53}{4} \text{ (rejected)} \quad \text{or} \quad n = 12$$

Hence, $n = 12$

Question 5.

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution:

Given: $a = 5, a_n = 45, S_n = 400$

$$S_n = \frac{n}{2}[a + a_n] \Rightarrow 400 = \frac{n}{2}[5 + 45]$$

$$\Rightarrow \quad 400 \times 2 = 50n \Rightarrow \frac{400 \times 2}{50} = n \Rightarrow n = 16$$

$$a_n = 45 \Rightarrow a + (n-1)d = 45 \Rightarrow 5 + (16-1)d = 45$$

$$\Rightarrow \quad 15d = 45 - 5 \Rightarrow 15d = 40 \Rightarrow d = \frac{40}{15} = \frac{8}{3}$$

Question 6.

The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

Here, $a = 17, a_n = 350, d = 9$

$$a_n = a + (n-1)d \Rightarrow 350 = 17 + (n-1)9$$

$$\Rightarrow \quad 350 - 17 = (n-1)9 \Rightarrow \frac{333}{9} = n - 1 \Rightarrow 37 + 1 = n \Rightarrow n = 38$$

$$S_{38} = \frac{38}{2}[17 + 350] = 19 \times 367 = 6973$$

Question 7.

Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Solution:

Given:

$$d = 7, a_{22} = 149$$

$$a_{22} = a + 21d = 149 \Rightarrow a + 21 \times 7 = 149$$

$$\Rightarrow \quad a + 147 = 149 \Rightarrow a = 149 - 147 = 2$$

$$S_{22} = \frac{22}{2}[a + a_{22}] = 11\,[2 + 149] = 11 \times 151 = 1661$$

Question 8.

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution:

Given: $a_2 = 14 \text{ and } a_3 = 18$

$\Rightarrow \quad a + d = 14 \quad ...(i) \quad \text{and } a + 2d = 18 \quad ...(ii)$

Substracting (*i*) and (*ii*), we get

$$a + 2d - a - d = 18 - 14 \Rightarrow d = 4$$

Since, $a + d = 14 \Rightarrow a + 4 = 14 \Rightarrow a = 14 - 4 \Rightarrow a = 10$

So, $S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1)4]$

$$S_{51} = \frac{51}{2}[20 + 200]$$

$$= \frac{51}{2} \times 220 = 51 \times 110 = 5610$$

Question 9.

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

Given: $S_7 = 49$ and $S_{17} = 289$

Since $S_7 = 49$

$\therefore \frac{7}{2}[2a + (7-1)d] = 49 \Rightarrow 2a + 6d = 49 \times \frac{2}{7} \Rightarrow 2a + 6d = 14$

$\Rightarrow a + 3d = 7$ ...(i)

and $S_{17} = 289 \Rightarrow \frac{17}{2}[2a + (17-1)d] = 289$

$\Rightarrow 2a + 16d = 289 \times \frac{2}{17} \Rightarrow 2a + 16d = 34$

or $a + 8d = 17$ ... (ii)

Subtracting (i) and (ii), we get

$$a + 8d - a - 3d = 17 - 7 \Rightarrow 5d = 10 \Rightarrow d = \frac{10}{5} = 2$$

$$a + 3d = 7 \Rightarrow a + 3 \times 2 = 7 \Rightarrow a = 7 - 6 = 1$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2 \times 1 + (n-1)2]$$

$$= \frac{n}{2}[2 + 2n - 2] = \frac{n}{2} \times 2n = n^2$$

Question 10.

Show that $a_1, a_2, \ldots\ldots a_n, \ldots\ldots$ form an AP where an is defined as below:

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Solution:

(i)
$$a_n = 3 + 4n$$
Putting $n = 1, 2, 3, \ldots\ldots$
$$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$$
$$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$$
$$d = a_2 - a_1 = 11 - 7 = 4$$
$$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$$
$$d = a_3 - a_2 = 15 - 11 = 4$$

Thus, the sequence 7, 11, 15, ... is an AP.

$$S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1) \times 4] = \frac{15}{2}[14 + 56] = \frac{15}{2} \times 70 = 525$$

(ii)
$$a_n = 9 - 5n$$
Putting $n = 1, 2, 3, \ldots$
$$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$$
$$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$$
$$d = a_2 - a_1 = -1 - 4 = -5$$
$$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$$
$$d = a_3 - a_2 = -6 - (-1) = -6 + 1 = -5$$

Thus, the sequence 4, –1, –6, ..., is an A.P.

$$a = 4, d = -1 - 4 = -5$$
$$S_{15} = \frac{15}{2}[2 \times 4 + (15 - 1)(-5)] = \frac{15}{2}[8 - 70]$$
$$= \frac{15}{2} \times (-62) = -465$$

Question 11.

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Solution:

$\because$

$$S_n = 4n - n^2$$
$$a_1 = S_1 = 4 \times 1 - 1^2 = 3$$

Now,

$$S_2 = 4 \times 2 - 2^2 = 4$$
$$a_2 = S_2 - a_1 = 4 - 3 = 1$$
$$d = a_2 - a_1 = 1 - 3 = -2$$
$$a_3 = a + 2d = 3 + 2(-2) = -1$$
$$a_{10} = a + 9d = 3 + 9(-2) = -15$$
$$a_n = a + (n-1)d = 3 - 2(n-1) = 5 - 2n$$

Question 12.

Find the sum of the first 40 positive integers divisible by 6.

Solution:

Let 6, 12, 18, ..., 240 be divisible by 6

$$a = 6, d = 12 - 6 = 6$$
$$a_{40} = 6 + (40 - 1) \times 6 = 240$$
$$S_{40} = \frac{40}{2}[a + a_{40}] = 20[6 + 240] = 20 \times 246 = 4920$$

Question 13.

Find the sum of the first 15 multiples of 8.

Solution:

Let 8, 16, 24, 32, ..., 120 are multiples of 8.

$$a = 8, d = 16 - 8 = 8,$$
$$a_{15} = 8 + (15 - 1) \times 8 = 8 + 112 = 120$$
$$S_{15} = \frac{15}{2}[a + a_{15}] = \frac{15}{2}[8 + 120] = \frac{15}{2} \times 128 = 960$$

Question 14.

Find the sum of the odd numbers between 0 and 50.

Solution:

Let odd numbers between 0 and 50 be 1, 3, 5, 7,........, 49.

Here, $a = 1, d = 3 - 1 = 2,$

$a_n = 49$

We have, $a_n = a + (n-1)d \Rightarrow a + (n-1)d = 49$

$\Rightarrow 1 + (n-1)2 = 49 \Rightarrow (n-1)2 = 49 - 1 \Rightarrow (n-1) = \frac{48}{2}$

$\Rightarrow n - 1 = 24 \Rightarrow n = 24 + 1 = 25$

$\therefore S_{25} = \frac{25}{2}[a + a_{25}] = \frac{25}{2}[1 + 49]$

$= \frac{25}{2} \times 50 = 25 \times 25 = 625$

Question 15.

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows:

₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc. the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Solution:

Given: $a = ₹\ 200, d = ₹\ 50, n = 30$

$S_{30} = \frac{n}{2}[2a + (n-1)d] = \frac{30}{2}[2a + (30-1)d] = 15[2 \times 200 + 29 \times 50]$

$= 15[400 + 1450] = 15 \times 1850 = ₹\ 27,750$

Question 16.

A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Solution:

Let 1st prize be of ₹ a

2nd prize be ₹ (a – 20) and

3rd prize be ₹ (a – 20 – 20) = ₹ (a – 40)

Then seven prizes are

₹ a, ₹ $(a - 20)$, ₹ $(a - 40)$, ..., ₹ $(a - 120)$ and $S_7 = 700$

$$a_1 = a, d = ₹ (a - 20 - a) = -₹ 20$$

We have, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_7 = \frac{7}{2}[2a + (7-1)d] \Rightarrow 700 = \frac{7}{2}[2a + 6d]$$

$$\Rightarrow \quad 700 = \frac{7}{2}[2a + 6(-20)] \Rightarrow 700 \times \frac{2}{7} = 2a - 120$$

$$\Rightarrow \quad 200 + 120 = 2a \Rightarrow 320 = 2a \Rightarrow a = \frac{320}{2} = ₹ 160$$

So, the seven prizes are ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

Question 17.

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, eg. a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Solution:

Let the trees be planted 1, 2, 3, 4, 5 , …… 12

Here, a = 1, d = 1, n = 12

Total number of trees planted by each section

S_{12} = 122 [2a + (n – 1) d] = 6 [2 x 1 + (12 – 1) x 1]

= 6 [2 + 11] = 6 x 13 = 78

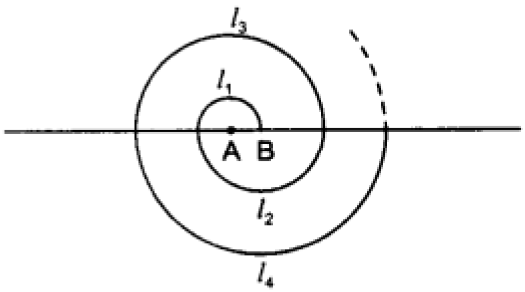
Total number of trees planted by 3 sections = 78 x 3 = 234

Question 18.

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles?

(Take π = 227)

[**Hint:** Length of successive semicircles is l_1, l_2, l_3, l_4, … with centres at A, B, respectively.]



Solution:

$R_1 = 0.5$ cm, $R_2 = 1.0$ cm, $R_3 = 1.5$ cm

$a = 0.5$ cm, $d = 1.0$ cm $- 0.5$ cm $= 0.5$ cm

Length of spiral $= 13$ consecutive semicircles $= \pi R_1 + \pi R_2 + \pi R_3 + \ldots\ldots + \pi R_{13}$

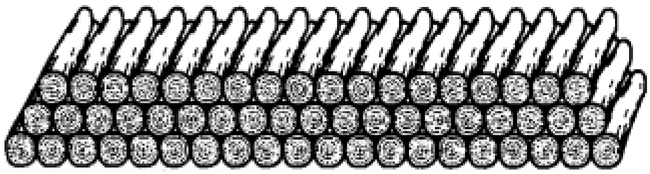
$= \pi [R_1 + R_2 + R_3 + \ldots\ldots R_{13}] = \pi [0.5 + 1.0 + 1.5 + \ldots. + 6.5]$

$= \pi\left\{\frac{13}{2}[2 \times 0.5 + (13-1)(0.5)]\right\} = \pi\left[\frac{13}{2}(1 + 12 \times 0.5)\right]$

$= \pi \times \left(\frac{13}{2} \times 7\right) = \frac{22}{7} \times \frac{13}{2} \times 7 = 143$ cm

Question 19.

200 logs are stacked in the following manner 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Solution:

Let number of rows be = n

and number of logs in the top be = a

Here, $a_n = 20, d = 1, S_n = 200$

We have, $a_n = a + (n-1)d \Rightarrow a + (n-1)1 = 20$

$\Rightarrow a + n = 20 + 1 \Rightarrow a + n = 21$

We also have, $S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 200 = \frac{n}{2}[a + a_n]$

$\Rightarrow 400 = n[a + 20] \Rightarrow na + 20n = 400$

$\Rightarrow n(21-n) + 20n = 400$ $[\because a + n = 21]$

$\Rightarrow 21n - n^2 + 20n = 400 \Rightarrow n^2 - 41n + 400 = 0$

$\Rightarrow n^2 - 25n - 16n + 400 = 0 \Rightarrow n(n-25) - 16(n-25) = 0$

$\Rightarrow (n-16)(n-25) = 0 \Rightarrow n = 16$ or $n = 25$

Taking $n = 16$,

$a + (n-1)d = 20 \Rightarrow a + (16-1)d = 20 \Rightarrow a + 15d = 20$

$\Rightarrow a + 15 \times 1 = 20 \Rightarrow a = 20 - 15 = 5$ [Accepted]

Taking $n = 25$,

$a + (25-1)d = 20 \Rightarrow a + 24 \times 1 = 20$

$\Rightarrow a = 20 - 24 = -4$ [Rejected]

Hence, number of rows are $n = 16$

and number of logs in the top row is $a = 5$.

Question 20.

In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig.) A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[**Hint:** To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 x 5 + 2 x (5 + 3)]

Solution:

Distance between the first potato and the bucket = 5 m

Distance between next 2 potatoes = 3 m each

So, series is 5 m, 8 m, 11 m,

Here, a = 5 m, d = (8 – 5) m = 3 m
Total distance travelled for 10 potatoes = 2 [5 + 8 + 11 + …….. + 10 terms]
= 2[102{2 x 5 + (10 – 1) 3}]
= 2[5{10 + 27}] = 2(37 x 5) = 37 x 10 = 370 m.

Ex 5.4 Class 10 Question 1.
Which term of the AP: 121, 117. 113, ….., is its first negative term?
Solution:

Here, $a = 121$, and $d = 117 - 121 = -4$

Let nth term be the Ist negative term.

Then $t_n = 121 + (n-1)(-4)$

$\Rightarrow \quad t_n = 121 - 4n + 4 \quad \Rightarrow \quad t_n = 125 - 4n$

Now, $125 - 4n < 0$ [∵ The term is negative]

$\Rightarrow \quad -4n < -125$

$\Rightarrow \quad n > \frac{125}{4} \quad \Rightarrow \quad n > 31\frac{1}{4}$

$\therefore \quad n = 32.$

Hence, the **32nd term** of the AP is its first negative term.

Ex 5.4 Class 10 NCERT Solutions Question 2.
The sum of the third term and the seventh term of an AP is 6 and their product is 8. Fibd the sum of first sixteen terms of the AP?
Solution:

Let a and d be the first term and common difference of an AP.

Then AP = $a, a + d, a + 2d, \ldots$

According to first condition:

$$t_3 + t_7 = 6$$

$$\Rightarrow \quad (a + 2d) + (a + 6d) = 6$$

$$\Rightarrow \quad a = 3 - 4d \quad \ldots(i)$$

and

$$t_3 \times t_7 = 8$$

$$\Rightarrow \quad (a + 2d)(a + 6d) = 8$$

$$\Rightarrow \quad [(3-4d)+2d][(3-4d)+6d] = 8$$

$$\Rightarrow \quad (3-2d)(3+2d) = 8$$

$$\Rightarrow \quad 9-4d^2 = 8$$

$$\Rightarrow \quad d^2 = \frac{1}{4} = \left(\frac{1}{2}\right)^2 \quad \Rightarrow \quad d = \pm\frac{1}{2}$$

When $d = \frac{1}{2}$, $a = 3 - 4 \times \frac{1}{2} = 1$ [Using (i)]

$$\therefore S_{16} = \frac{16}{2}\left[2 \times 1 + (16-1) \times \frac{1}{2}\right]$$

$$= 8\left[2 + \frac{15}{2}\right] = 8\left[\frac{19}{2}\right] = 76.$$

When $d = -\frac{1}{2}$, $a = 3 - 4 \times \left(\frac{-1}{2}\right) = 5$

[Using (i)]

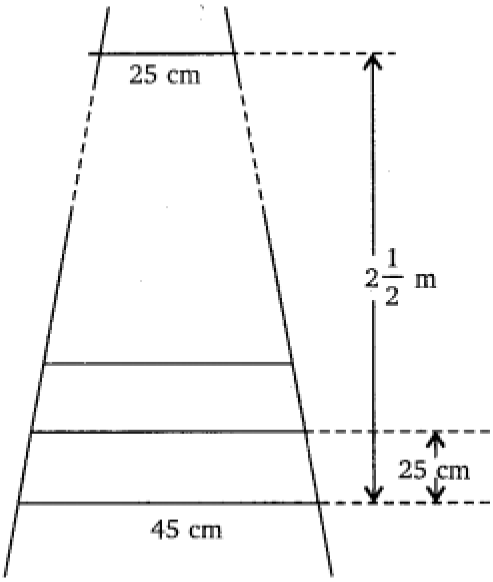
$$\therefore S_{16} = \frac{16}{2}\left[2 \times 5 + (16-1) \times \frac{-1}{2}\right]$$

$$= 8\left[10 + \left(\frac{-15}{2}\right)\right] = 8\left[10 - \frac{15}{2}\right]$$

$$= 8 \times \frac{5}{2} = 20.$$

Hence, the sum of first sixteen terms of the given AP is either **20** or **76**.

Class 10 Ex 5.4 Question 3.
A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 at the top. If the top and the bottom rungs are $2\frac{1}{2}$m apart, what is the length of the wood required for the rungs?



Solution:

Here, length of rung at the bottom $(a) = 45$ cm

and length of rung at the top $(l) = 25$ cm.

As the top and the bottom rungs are $\frac{5}{2}$m apart,

$\therefore$ Number of rungs $= \frac{250}{25} + 1 = 11.$

[5 m = 250 cm]

Sum of lengths of 11 rungs $(S_n) = \frac{n}{2}(a + l)$

$\Rightarrow S_{11} = \frac{11}{2}(45 + 25) = 11 \times 35 = 385$ cm.

Hence, the required length of wood is **385 cm**.

Ex 5.4 Class 10 Question 4.

The houses of a row are numbered consecutively from 1 to 49. Show that there is value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

Solution:

Here, $a = 1$, $d = 1$ and $x = 49$

$\therefore$ Sum of the numbers of the houses preceding the house numbered $x = S_{x-1}$.

According to the question, we have

$$S_{x-1} = S_{49} - S_x$$

$$\Rightarrow \frac{x-1}{2}[2a + (x-1-1)d] = \frac{49}{2}[2a + 48d] - \frac{x}{2}[2a + (x-1)d]$$

$$\Rightarrow \frac{x-1}{2}[2a + (x-2)d] = \frac{49}{2}[2a + 48d] - \frac{x}{2}[2a + (x-1)d]$$

$$\Rightarrow x - 1[2a + (x-2)d] = 49[2a + 48d] - x[2a + xd - d]$$

Substituting the value of a and d, we get:

$$(x-1)(2 + x - 2) = 49(2 + 48) - x(2 + x - 1)$$

$$\Rightarrow (x-1)x = 49 \times 50 - x(x+1)$$

$$\Rightarrow x(x-1) + x(x+1) = 49 \times 50$$

$$\Rightarrow 2x^2 = 2450 \Rightarrow x^2 = \frac{2450}{2}$$

$$= 1225 = (35)^2$$

$$\Rightarrow x = \mathbf{35}$$