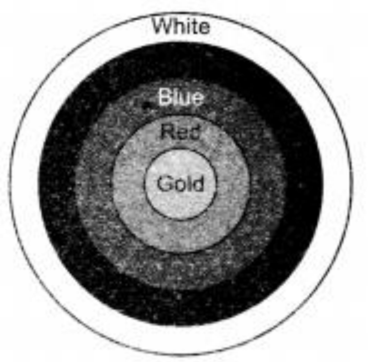
NCERT Solutions Of Chapter 12 – Areas related to circles

Ex 12.1 Class 10 Question 1.
The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
Solution:
Given: radius of 1st circle (R_1) = 19 cm
∴ Circumference of 1st circle = $2\pi R_1 = 2\pi(19)$ cm
Radius of 2nd circle (R_2) = 9 cm
∴ Circumference of 2nd circle = $2\pi R_2 = 2\pi(9)$ cm
Let radius of 3rd circle be R_3
Circumference of 3rd circle = $2\pi R_3$
According to question,
$2\pi R_1 + 2\pi R_2 = 2\pi R_3$
$\Rightarrow 2\pi(R_1 + R_2) = 2\pi R_3$
$\Rightarrow R_1 + R_2 = R_3$
$\Rightarrow 19 + 9 = R_3$
$\Rightarrow R_3 = 28$ cm

Class 10 Maths Chapter 12.1 Question 2.
The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
Solution:
Given: radius of 1st circle (R_1) = 8 cm
Area of 1st circle = $\pi R_1^2 = \pi(8)^2 cm^2$
Radius of 2nd circle (R_2) = 6 cm
Area of 2nd circle = $\pi R_2^2 = \pi(6)^2$ cm^2
Let radius of 3rd circle be R_3
Area of 3rd circle = πR_3^2
According to question,
$\pi R_1^2 + \pi R_2^2 - \pi R_3^2$
$\Rightarrow R_1^2 + R_2^2 = R_3^2 \Rightarrow (8)^2 + i6)^2 - R_3^2$
$\Rightarrow 64 + 36 = R_3^2 \Rightarrow R_3 = \sqrt{100}$ = 10 cm

Areas Related To Circles Exercise 12.1 Question 3.
The figure depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue,
Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.



Solution:

Diameter of the region representing gold score is 21 cm

⇒ Radius of the region representing gold region = $\frac{21}{2}$ cm

$\therefore$ Area of the **gold** region only $= \pi r^2 = \pi\left(\frac{21}{2}\right)^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{11 \times 3 \times 21}{2}$

Area of the **gold** region only $=$ $\boxed{346.5 \text{ cm}^2}$

Width of the **red** circle $= 10.5 \text{ cm} = \frac{21}{2} \text{cm}$

Radius of the **red** region including **gold** region

$=$ Radius of the **gold** region + Width of the **red** region

$= \frac{21}{2} + \frac{21}{2} = 21 \text{ cm}$

$\therefore$ Area of the **red** region including **gold** region

$= \pi(21)^2 \text{cm}^2 = \frac{22}{7} \times 21 \times 21 = 22 \times 3 \times 21 = 1386 \text{ cm}^2$

Area of the **red** region only $=$ Area of the **red** region including **gold** region − Area of the **gold** region

$= 1386 \text{ cm}^2 - 346.5 \text{ cm}^2$

Area of the **red** region only $=$ $\boxed{1039.5 \text{ cm}^2}$

Width of the **blue** circle $= 10.5 \text{ cm} = \frac{21}{2} \text{cm}$

$\therefore$ Radius up to **blue** region $=$ Radius of the **gold region** + width of the **red** region + width of the **blue** region

$= \frac{21}{2}\text{cm} + \frac{21}{2}\text{cm} + \frac{21}{2}\text{cm} = \frac{21+21+21}{2}\text{cm} = \frac{63}{2}\text{cm}$

Area of the **blue** region including **red** and **gold** region

$= \frac{22}{7}\left(\frac{63}{2}\right)^2 = \frac{22}{7} \times \frac{63}{2} \times \frac{63}{2} = 3118.5 \text{ cm}^2$

Area of the **blue** region only $=$ Area of the (**blue** + **red** + **gold**) region − Area of the (**red** + **gold**) region

$= 3118.5 \text{ cm}^2 - 1386 \text{ cm}^2 =$ $\boxed{1732.5 \text{ cm}^2}$

Radius up to **black** region $= \frac{21}{2}\text{cm} + \frac{21}{2}\text{cm} + \frac{21}{2}\text{cm} + \frac{21}{2}\text{cm} = \frac{84}{2} \text{ cm} = 42 \text{ cm}$

Area including (**black** + **blue** + **red** + **gold**) region $= \frac{22}{7} \times 42 \times 42 = 5544 \text{ cm}^2$

Area of the (**blue** + **red** + **gold**) region $= 3118.5 \text{ cm}^2$

Area of the **black** region only = Area of the (**black** + **blue** + **red** + **gold**) region − Area of the (**blue** + **red** + **gold**) region

Area of the **black** region only $= 5544 \text{ cm}^2 - 3118.5 \text{ cm}^2 =$ $\boxed{2425.5 \text{ cm}^2}$

Radius up to **white** boundary = Radius up to **black** region + width of the **white** region

$= 42 \text{ cm} + \frac{21}{2} \text{ cm} = \frac{105}{2} \text{ cm}$

Area of the (**white + black + blue + red + gold**) region

$$= \frac{22}{7} \times \left(\frac{105}{2}\right)^2 cm^2 = \frac{22}{7} \times \frac{105 \times 105}{4} cm^2 = \boxed{8662.5\ cm^2}$$

Area of the **white** region only = Area of the (**white + black + blue + red + gold**) region

– Area of the (**black + blue + red + gold**) region

$$= 8662.5\ cm^2 - 5544\ cm^2 = \boxed{3118.5\ cm^2}$$

Exercise 12.1 Class 10 Question 4.

The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Solution:

Given: diameter of the wheels of the car = 80 cm

⇒ Radius of the wheel of the car = 802 = 40 cm

Circumference of the wheel = 2πr = 2 x 222 x 40 cm

Speed of the car = 66 km/h

Distance covered in 10 minutes =66x1060 = 11 km

= 11 x 1000 x 100 cm = 11,00,000 cm

$$\therefore \text{ Number of revolutions} = \frac{\text{Total distance in 10 minutes}}{\text{Circumference of the wheel}} = \frac{1100000}{2 \times \frac{22}{7} \times 40} = 4375$$

Ex 12.1 Class 10 Maths Solution Question 5.

Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(a) 2 units

(b) n units

(c) 4 units

(d) 7 units

Solution:

Let radius of the circle = r units

Perimeter of the circle = 2πr

Area of the circle = πr^2

According to question,

Perimeter of the circle = Area of the circle

⇒ $2\pi r = \pi r^2$

⇒ r = 2 units

Hence, option (a) is correct.

Ex 12.2

Question 1.
Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.
Solution:
Radius of the sector (r) = 6 cm
Central angle of the sector = 60°

$$\text{Area of the sector of circle} = \frac{\pi r^2\theta}{360^\circ} = \frac{22\times6\times6\times60}{7\times360} = \frac{132}{7}\text{ cm}^2$$

Question 2.
Find the area of a quadrant of a circle whose circumference is 22 cm.
Solution:
Let radius of the circle = r
∴ Circumference of the circle = 2πr

According to question,

$$2\pi r = 22\text{ cm}$$

$$\Rightarrow \quad 2\times\frac{22}{7}\times r = 22 \Rightarrow r = \frac{22\times7}{2\times22} = \frac{7}{2}\text{cm}$$

$$\text{Area of quadrant of the circle} = \frac{\pi r^2\theta}{360^\circ} = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{90^\circ}{360^\circ} = \frac{22\times7}{2\times2\times4} = \frac{77}{8}\text{ cm}^2$$

Question 3.
The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.
Solution:
Length of minute hand of the clock = 14 cm

$$\text{Angle swept in 5 minutes} = \frac{360^\circ}{60}\times5 = 30^\circ$$

$$\text{Area swept in 5 minutes} = \frac{22}{7}\times\frac{14\times14\times30^\circ}{360^\circ} = \frac{11\times2\times14}{6} = \frac{11\times14}{3} = \frac{154}{3}\text{ cm}^2$$

Question 4.
A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:
(i) minor segment
(ii) major segment (Use ? = 3.14)
Solution:
Given: radius of the circle = 10 cm
Angle subtended by chord at centre = 90°
(i) Area of the minor segment

Area of the minor segment

= Area of the sector OAPB

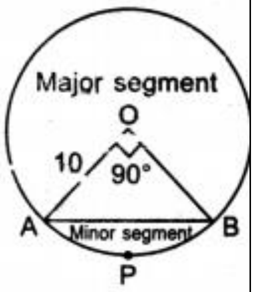
– Area of ΔAOB formed with radius and chord

$$= \frac{\pi r^2\theta}{360^\circ} - \frac{1}{2}r^2\sin\theta$$

$$= 3.14 \times \frac{10 \times 10 \times 90^\circ}{360^\circ} - \frac{1}{2} \times 10 \times 10 \times \sin 90^\circ$$

$$= 3.14 \times \frac{10 \times 10}{4} - \frac{1}{2} \times 10 \times 10$$

$$= 3.14 \times 25 - 50 = 78.5 - 50 = 28.5 \text{ cm}^2$$



(ii) Area of the major segment = Area of the circle – Area of the minor segment

= πr^2 – 28.5 = 3.14 x 10 x 10-28.5

= 314-28.5 = 285.5 cm^2

Question 5.

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) length of the arc.

(ii) area of the sector formed by the arc.

(iii) area of the segment formed by the corresponding chord.

Solution:

Radius of the circle = 21 cm

Angle at the centre = 60°

(*i*) Length of the arc $= \frac{\pi r\theta}{180^\circ} = \frac{22}{7} \times \frac{21 \times 60^\circ}{180^\circ} = 22 \text{ cm}$

(*ii*) Area of the sector formed by the arc

$$= \frac{\pi r^2\theta}{360^\circ} = \frac{22 \times 21 \times 21 \times 60^\circ}{7 \times 360^\circ} = \frac{22 \times 3 \times 21}{6}$$

$$= 11 \times 21 = 231 \text{ cm}^2$$

(*iii*) Area of the segment formed by the corresponding chord = area of the sector – area of the Δ formed between chord and radius of the circle

$$= 231 - \left(\frac{1}{2}r^2\sin\theta\right) = 231 - \frac{1}{2} \times (21)^2 \times \sin 60^\circ$$

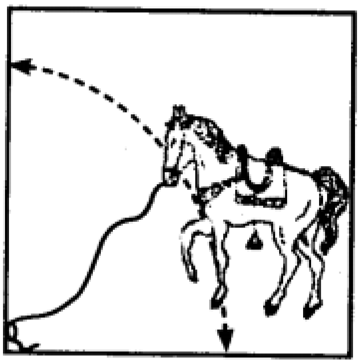
$$= 231 - \frac{1}{2} \times 441 \times \frac{\sqrt{3}}{2} = 231 - \frac{441\sqrt{3}}{4}$$

$$= 231 - 190.95 = 40.05 \text{ cm}^2$$

Question 6.

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the

areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73) **Solution:** Radius of the circle = 15 cm Angle subtended by chord at centre = 60° Area of the sector = $\frac{\pi r^2 \theta}{360^\circ}$ = 3.14 x $\frac{15 \times 15 \times 60^\circ}{360^\circ}$ = 117.75 cm^2 Area of the triangle formed by radii and chord = $\frac{1}{2} r^2 \theta$ = $\frac{1}{2}(15)^2$ sin 60° = $\frac{1}{2}$ x 15 x 15 x $\frac{\sqrt{3}}{2}$ = 97.31 cm^2 Area of the minor segment = Area of the sector – Area of the triangle formed by radii and chord = 117.75 – 97.31 = 20.44 cm^2 Area of the circle = πr^2 = 3.14 x 15 x 15 = 706.5 cm^2 Area of the circle – Area of the minor segment = 706.5 – 20.44 = 686.06 cm^2
Question 7. A chord of a circle of the radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73). **Solution:** Radius of the circle = 12 cm Angle subtended by chord at centre = 120° Area of the sector = $\frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times 12 \times 12 \times 120^\circ}{360^\circ} = 150.72\ cm^2$ Area of triangle formed by radii and chord = $\frac{1}{2} r^2 \sin\theta = \frac{1}{2}(12)^2 \sin 120^\circ$ $= \frac{1}{2} \times \frac{12 \times 12 \times \sqrt{3}}{2}$ $\left[\sin 120^\circ = \frac{\sqrt{3}}{2}\right]$ $= 3 \times 12 \times 1.73 = 62.28\ cm^2$ Area of the corresponding segment = Area of the sector – Area of A formed by radii and chord = 150.72 – 62.28 cm^2 = 88.44 cm^2
Question 8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find



(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use n = 3.14)

Solution:

(i) Length of the rope = Radius of the sector grazed by horse = 5 m
Here, angle of the sector = 90°

$$\text{Area of the field that horse can graze} = \frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times 5 \times 5 \times 90^\circ}{360^\circ}$$

$$= \frac{3.14 \times 5 \times 5}{4} = 19.625 \text{ cm}^2$$

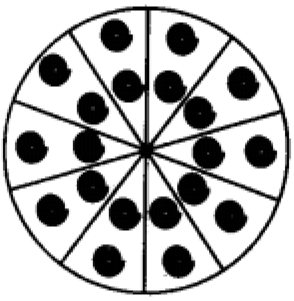
Length of the rope is increased from 5 m to 10 m
New radius of sector grazed by horse = 10 m

$$\therefore \quad \text{Area grazed by horse} = \frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times 10 \times 10 \times 90^\circ}{360^\circ} = 78.5 \text{ cm}^2$$

$$\text{Area increased} = 78.5 - 19.625 = 58.875 \text{ cm}^2.$$

Question 9.

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure.



Find:

(i) the total length of the silver wire required.
(ii) the area of each sector of the brooch.

Solution:

Length of one diameter = 35 mm
Total length of 5 diameters = 5 x 35 mm = 175 mm
Circumference of the circle = 2π = 2 x 227 x 352 = 110 mm
(i) Total length of the wire used = length of 5 diameters + circumference of brooch
= 175 + 110 = 285 mm

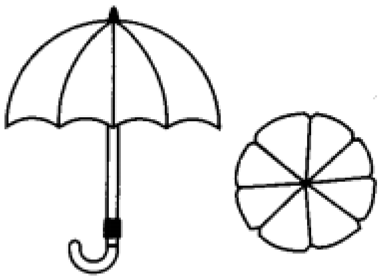
(ii) Total sectors are 10.

$$\text{Central angles for each sector} = \frac{\text{central angle of the circle}}{\text{number of the sectors}} = \frac{360^\circ}{10} = 36^\circ$$

$$\text{Area swept by each sector} = \frac{\pi r^2\theta}{360^\circ} = \frac{22 \times 35 \times 35 \times 36^\circ}{7 \times 2 \times 2 \times 360^\circ} = \frac{385}{4}\ \text{mm}^2$$

Question 10.

An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Solution:

Radius of the circle = 45 cm
Number of ribs = 8

$$\text{Angle between two consecutive ribs} = \frac{\text{central angle of the circle}}{\text{number of the sectors (ribs)}} = \frac{360^\circ}{8} = 45^\circ$$

Area between two consecutive ribs = Area of one sector of the circle

$$= \frac{\pi r^2\theta}{360^\circ} = \frac{22}{7} \times \frac{45 \times 45 \times 45^\circ}{360^\circ}$$

$$= \frac{11 \times 45 \times 9 \times 5}{7 \times 4} = \frac{22275}{28}\ \text{cm}^2$$

Question 11.

A car has two wipers which do not overlap.

Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution:

Given: length of blade of wiper = radius of sector sweep by blade = 25 cm

Area cleaned by each sweep of the blade = area of sector sweep by blade

Angle of the sector formed by blade of wiper =115°

$$\text{Area of the sector formed} = \frac{\pi r^2 \theta}{360^\circ}$$

$$= \frac{22}{7} \times \frac{25 \times 25 \times 115^\circ}{360^\circ} = \frac{11 \times 25 \times 25 \times 23}{7 \times 36}$$

Total area cleaned at each sweep of the blade = 2 (area cleaned by one wiper)

$$= \frac{2 \times 11 \times 25 \times 25 \times 23}{7 \times 36}$$

$$= \frac{11 \times 25 \times 25 \times 23}{7 \times 18} = \frac{158125}{126} \text{ cm}^2$$

Question 12.

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

Solution:

Angle of the sector = 80°

Distance covered = 16.5 km

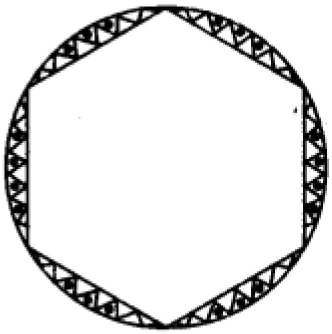
Radius of the sector formed = 16.5 km

$$\text{Area of the sea over which ships are warned} = \text{Area of the sector} = \frac{\pi r^2 \theta}{360^\circ}$$

$$= \frac{3.14 \times 16.5 \times 16.5 \times 80^\circ}{360^\circ} = 189.97 \text{ km}^2$$

Question 13.

A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm². (Use $3-\sqrt{}$ = 1.7)

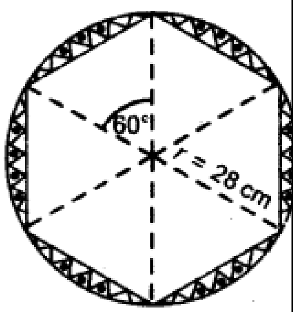


Solution:

Radius of the cover = 28 cm

∵ There are six equal designs

$\therefore$ Angle at centre $= \dfrac{360^\circ}{6} = 60^\circ$



Area of the sector $= \dfrac{\pi r^2 \theta}{360^\circ}$

$= \dfrac{22}{7} \times \dfrac{28 \times 28 \times 60^\circ}{360^\circ}$

$= \dfrac{22 \times 28 \times 2}{3}$ cm^2

$= 410.66$ cm^2

Area of one triangle formed between radii and chord

$= \dfrac{1}{2} r^2 \sin\theta = \dfrac{1}{2} \times 28 \times 28 \times \sin 60^\circ$

$= \dfrac{1}{2} \times 28 \times 28 \times \dfrac{\sqrt{3}}{2}$

$= 7 \times 28 \times 1.7 = 333.2$ cm^2

Area of one minor segment = Area of the sector – Area of the triangle formed between radii and chord

$= 410.66 - 333.2 = 77.46$ cm^2

Total area covered in design = 6 (area of one segment)

$= 6 \times 77.46 = 464.76$ cm^2

Cost of making the design = Rate per cm^2 × Total area covered

$= 0.35 \times 464.76 = ₹\ 162.66$

Question 14.

Tick the correct answer in the following: Area of a sector of angle p (in degrees) of a circle with radius R is

(a) $\frac{p}{180^\circ} \times 2\pi R$

(b) $\frac{p}{180^\circ} \times \pi R^2$

(c) $\frac{p}{360^\circ} \times 2\pi R$

(d) p720∘ × 2πR2

Solution:

Sector angle isp in degrees

Radius of the circle = R

Area of the sector = πR2p3616 = (πR2p)2720∘

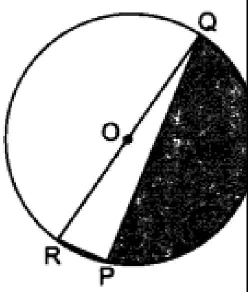
= p720∘ × 2πR2

Ex 12.3

Question 1.

Find the area of the shaded region in the given figure, if PQ = 24cm, PR = 7cm and O is the centre of the circle.

Solution:



In ΔPQR, $\angle QPR = 90°$ (angle in semicircle)

and $QR^2 = PQ^2 + RP^2$

(as Δ QPR is a right angled triangle)

$\therefore \quad QR^2 = (24)^2 + (7)^2 = 576 + 49$

$\Rightarrow \quad QR^2 = 625 \text{ cm}^2$

$\Rightarrow \quad QR = \sqrt{625} \Rightarrow QR = 25$

Area of the shaded region = Area of the semicircle – Area of the triangle

$$= \frac{1}{2}\pi r^2 - \text{area of } \Delta PQR = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2 - \frac{1}{2} \times PR \times QP$$

$$= \frac{22}{7} \times \frac{25 \times 25}{2 \times 4} - \frac{1}{2} \times 7 \times 24 = \frac{22 \times 625}{28 \times 2} - 84 = \frac{13750 - 4704}{56}$$

$$= \frac{9046}{56} = \frac{4523}{28} = 161.54 \text{ cm}^2$$

Question 2.

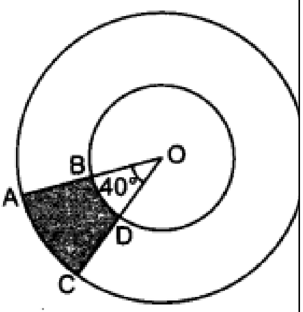
Find the area of the shaded region in the given figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 400.

Solution:

∠AOC = 40° (given)

Radius of the sector AOC = 14 cm

$$\text{Area of the sector AOC} = \frac{\pi r^2 \theta}{360^\circ} = \frac{\pi (14)^2 \times 40^\circ}{360^\circ}$$
$$= \frac{22}{7} \times \frac{14 \times 14 \times 40^\circ}{360^\circ}$$
$$= \frac{22 \times 2 \times 14 \times 1}{9} = \frac{616}{9} \text{ cm}^2$$



$$\text{Radius of the sector BOD} = 7 \text{ cm}$$

$$\text{Area of the sector BOD} = \frac{\pi r^2 \theta}{360^\circ} = \frac{22 \times 7 \times 7 \times 40^\circ}{7 \times 360^\circ} = \frac{22 \times 7 \times 1}{9} = \frac{154}{9} \text{ cm}^2$$

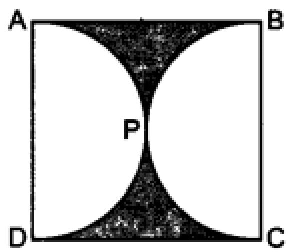
$$\text{Area of the shaded region} = \text{Area of the sector AOC} - \text{Area of the sector BOD}$$
$$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9} = \frac{154}{3} \text{ cm}^2$$

Question 3.

Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

Solution:

ABCD is a square



Given: side of the square = 14 cm

∴ Area of the square = $(\text{side})^2 = (14)^2 = 196 \text{ cm}^2$

Radius of the semicircle APD = 12(side of square) = 12 x 14 = 7 cm

Area of the semicircle APD = $12\ \pi r^2 = 12 \times 227 \times 7 \times 7 = 11 \times 7 = 77 \text{cm}^2$

Similarly, area of the semicircle BPC = 77 cm^2

Total area of both the semicircles = $77 + 77 = 154 \text{ cm}^2$

Area of the shaded region = Area of square – area of both semicircles

$= 196 - 154 = 42 \text{ cm}^2$

Question 4.

Find the area of the shaded region in the figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

Solution:

Area of the equilateral triangle OAB

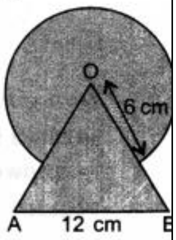
$$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(12)^2 = \frac{\sqrt{3} \times 12 \times 12}{4} = 36\sqrt{3} \text{ cm}^2$$

$\angle AOB = 60°$ (angle of the equilateral triangle)

Exterior $\angle AOB = 360° - 60° = 300°$

Area of the sector of circle having radius 6 cm and sector angle 300°

$$= \frac{\pi r^2 \theta}{360°} = \frac{22}{7} \times \frac{6 \times 6 \times 300°}{360°} = \frac{22 \times 30}{7} = \frac{660}{7} \text{ cm}^2$$

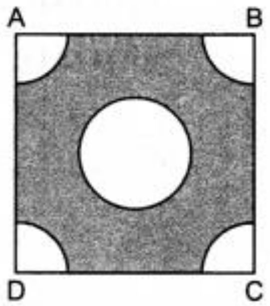


Total area of the shaded part = area of the triangle + area of the sector

$$= 36\sqrt{3} + \frac{660}{7} = \left(\frac{660}{7} + 36\sqrt{3}\right) \text{ cm}^2$$

Question 5.

From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the figure. Find the area of the remaining portion of the square.



Solution:

Given: side of the square ABCD = 4 cm

Area of the square ABCD = 4 x 4 = 16 cm²

Radius of the quadrant at corner = 1 cm

$$\text{Area of the quadrant at each corner} = \frac{\pi r^2 \theta}{360°} = \frac{22}{7} \times \frac{1 \times 1 \times 90°}{360°} = \frac{22}{28} \text{ cm}^2$$

$$\text{Now, area of the 4 sectors at each corner} = \frac{4 \times 22}{28} = \frac{22}{7} \text{ cm}^2$$

Area of the circle at the centre of the square having (radius $= \frac{2}{2} = 1$ cm)

$$= \pi r^2 = \frac{22}{7}(1)^2 = \frac{22}{7} \text{ cm}^2$$

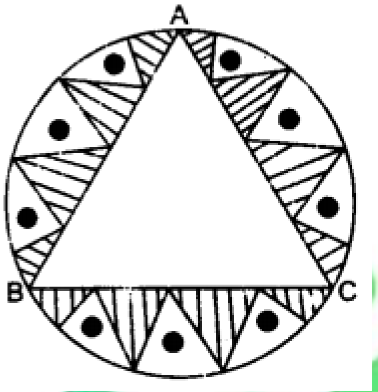
Total area to be cut out from square = Area of the 4 sectors + Area of the circle at the centre

$$= \frac{22}{7} + \frac{22}{7} = \frac{44}{7} \text{ cm}^2$$

∴ Area of the remaining portion = Area of the square – Area to be cut from square

= 16 – (447) = 16 – 447
= 112−447 = 687cm²

Question 6.
In a circular table cover of the radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the figure. Find the area of the design (shaded region).



Solution:
Radius of the circle(r) = 32cm
Area of the circle = πr^2
= 227 × 32 × 32 = 225287cm²
∴ An equilateral triangle is formed in the circle as shown
Angle subtended by et ch side at centre

$$= \frac{\text{Angle of the centre}}{\text{Total number of the sides}}$$

$$= \frac{360°}{3} = 120°$$

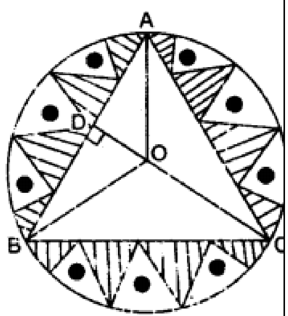
In ΔOAB, as we know that perpendicular drawn from centre on chord bisect the chord

In ΔOAD, $\angle AOD = \frac{120°}{2} = 60°$

$\Rightarrow$ $\sin 60° = \frac{AD}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{32}$

$\Rightarrow$ $\frac{32\sqrt{3}}{2} = AD \Rightarrow 16\sqrt{3} = AD$

$AB = 2AD = 2(16\sqrt{3}) = 32\sqrt{3}$ cm



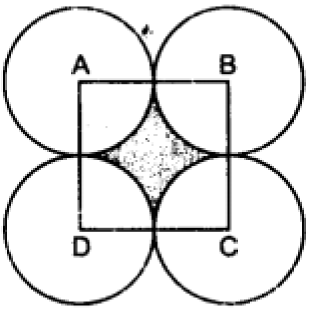
Area of the equilateral triangle ABC $= \frac{\sqrt{3}}{4}(32\sqrt{3})^2 = \frac{\sqrt{3} \times 32 \times 32 \times 3}{4}$

$= 8 \times 32 \times 3\sqrt{3} = 768\sqrt{3}$ cm²

Area of the shaded design = Area of the circle – Area of the equilateral triangle

$= \frac{22528}{7} - 768\sqrt{3}$ cm²

Question 7.
In the figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



Solution:
Side of the square ABCD = 14 cm
Area of the squat e = (side)² = 14 x 14 = 196²

Radius of the circle drawn at vertex

$$= \frac{1}{2} \text{(side of square)} = \frac{1}{2} \times 14 = 7 \text{ cm}$$

Area of the sector (quadrant) formed at each vertex

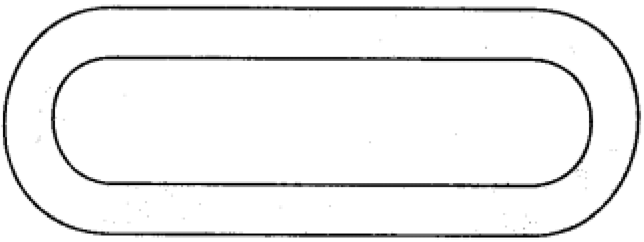
$$= \frac{\pi r^2 \theta}{360^\circ} = \frac{22 \times 7 \times 7 \times 90^\circ}{7 \times 360^\circ} = \frac{22 \times 7}{4} = \frac{77}{2} \text{ cm}^2$$

Total area of the 4 quadrants $= \frac{4 \times 77}{2} = 154 \text{ cm}^2$

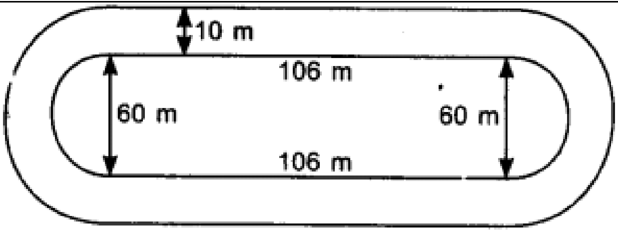
Area of the shaded region = area of the square – area of the 4 sectors

$= 196 - 154 = 42 \text{ cm}^2$

Question 8.
The given figure depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:



(i) the distance around the track along its inner edge.
(ii) the area of the track.
Solution:



(i) Circumference of the semicircular ends at both sides = $\frac{2\pi r}{2} = \pi r$

Radius of the circular ends on both sides $r = \frac{60}{2} = 30$ cm

Circumference of the semicircular ends = $\frac{22}{7} \times 30 = \frac{660}{7}$ cm

Total length of the semicircular ends = $\frac{2 \times 660}{7} = \frac{1320}{7}$ cm

Total length of the inner racing track = $106 + 106 + \frac{1320}{7} = 212 + \frac{1320}{7}$

$= \frac{1484 + 1320}{7} = \frac{2804}{7}$ cm

(ii) Radius of the outer semicircular end = 30 + 10 = 40 cm

Area of the outer semicircular ends

$= \frac{1}{2}(\pi r^2) = \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{11 \times 40 \times 40}{7} = \frac{17600}{7}$ cm

Area of the inner semicircular ends

$= \frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{11 \times 900}{7} = \frac{9900}{7}$ cm^2

Area of the track between semicircular ends

$= \frac{17600}{7} - \frac{9900}{7} = \frac{7700}{7}$ cm^2 = 1100 cm^2

Area of the tracks at both semicircular ends = 2 x 1100 = 2200 cm^2

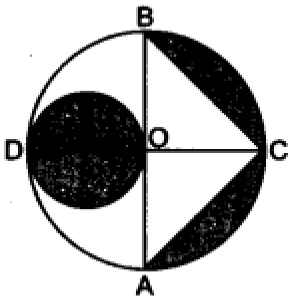
Area of the 2 rectangular portions = 2 x l x h = 2 x 106 x 10 = 2120 cm^2

Total area of the track = area of the track at semicircular ends + area of the rectangular portions

= 2200 + 2120 = 4320 cm^2

Question 9.

In the figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



Solution:
Given: OA = 7 cm
Radius of the semicircle ABC = OA = 7 cm
Area of the semicircle ABC = 12πr² = 12 x 227 x 7 x 7 = 11 x 7 = 77 cm²
Diameter AB = 2(OA) = 2 x 7 = 14 and OA = OC = 7 cm (radius)

$$\text{Area of the } \Delta ABC = \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 14 \times 7 = 49 \text{ cm}^2$$

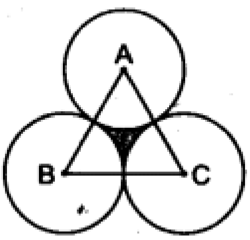
$$\text{Area of the circle having diameter } (OD = 7 \text{ cm}) = \pi\left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{7 \times 7}{2 \times 2} = \frac{11}{2} \times 7 = \frac{77}{2} \text{ cm}^2$$

$$\text{Area of the circle having radius } (OA = 7 \text{ cm}) = \pi(7)^2 = \frac{22}{7} \times 49 = 22 \times 7 = 154 \text{ cm}^2$$

$$\text{Area of the shaded region} = \{\text{Area of the circle having diameter OD} + (\text{Area of the semicircle ABC} - \text{Area of the } \Delta ABC)$$

$$= \frac{77}{2} + (77 - 49) = \frac{77}{2} + 28 = \frac{77 + 56}{2} = \frac{133}{2} = 66.5 \text{ cm}$$

Question 10.
The area of an equilateral triangle ABC is 17320.5 cm². With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see figure). Find the area of the shaded region.
(Use π = 3.14 and 3–√ = 1.73205).



Solution:
Given: area of an equilateral triangle ABC = 17320.5 cm²
Let side of the triangle AB'C be 'a'
∴ Area of the ΔABC = 3√4a2
3√4a2 = 17320.5

$$\Rightarrow \qquad a^2 = \frac{17320.5 \times 4}{\sqrt{3}} \Rightarrow a^2 = \frac{17320.5 \times 4}{1.73205} \text{ (Here } \sqrt{3} = 1.73205)$$

$$\Rightarrow \qquad a^2 = 40000 \qquad \Rightarrow a = 200 \text{ cm}$$

Radius of the circle drawn at each vertex

$$= \frac{1}{2}(\text{side of the equilateral triangle})$$

$$= \frac{1}{2} \times 200 = 100 \text{ cm}$$

Area of the sector formed at each vertex having radius 100 cm and sector angle 60°

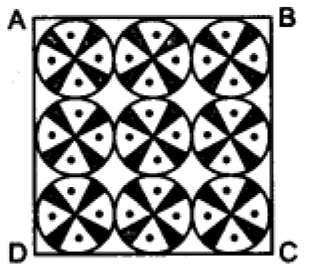
$$= 3.14 \times \frac{100 \times 100 \times 60°}{360°} = \frac{3.14 \times 100 \times 100}{6} = \frac{31400}{6}$$

Now, area of the all 3 sectors $= \frac{3 \times 31400}{6} = 15700 \text{ cm}^2$

Now,
area of the all 3 sectors = 3×314006 = 15700 cm²
∴ Area of the shaded portion = Area of the equilateral triangle
– Area of the three sectors formed at each vertex)
= 17320.5 – 15700 = 1620.5 cm²

Question 11.

On a square handkerchief, nine circular designs each of the radius 7 cm are made (see figure). Find the area of the remaining portion of the handkerchief.

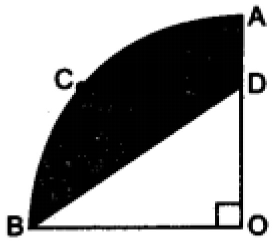


Solution:

Radius of the one circular design = 7 cm
Area of the one circular design = πr^2 = 227 x 7 x 7 = 154 cm²
Now, area of the 9 circular designs = 9 x 154 = 1386 cm²
Diameter of the circular design = 7 x 2 = 14 cm
Side of the square = 3(diameter of one circle) = 3 x 14 = 42 cm
Area of the square = 42 x 42 = 1764 cm²
Area of the remaining portion of handkerchief
= Area of the square – (Area of the 9 circular designs)
= 1764 – 1386
= 378 cm²

Question 12.

In the figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB, (ii) shaded region.



Solution:

(i) Radius of the quadrant OACB = 3.5 cm

$$\text{Area of the quadrant OACB} = \frac{\pi r^2\theta}{360^\circ} = \frac{22}{7} \times \frac{3.5 \times 3.5 \times 90^\circ}{360^\circ}$$

$$= \frac{22 \times 35 \times 35 \times 90^\circ}{7 \times 360^\circ \times 100} = \frac{77}{8}\ \text{cm}^2$$

(ii) OD = 2cm and OB = 3.5 cm

$\therefore$ $$\text{Area of the } \Delta\text{OBD} = \frac{1}{2} \times \text{OB} \times \text{OD} = \frac{1}{2} \times 3.5 \times 2 = 3.5\ \text{cm}^2$$

$$\text{Area of the shaded region} = \text{Area of the quadrant} - \text{Area of the } \Delta\text{OBD}$$

$$= \frac{77}{8} - \frac{35}{10} = \frac{77}{8} - \frac{7}{2} = \frac{77-28}{8} = \frac{49}{8}\ \text{cm}^2$$

Question 13.

In the figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use re = 3.14)

Solution:

Given: side of the square OABC = OA = 20 cm

Area of the square = 20 x 20 = 400 cm²

(Diagonal of the square)² = (side of the square)² + (side of the square)² (By pythagoras theorem)

Diagonal of the square = 2−√ x (side of the square)

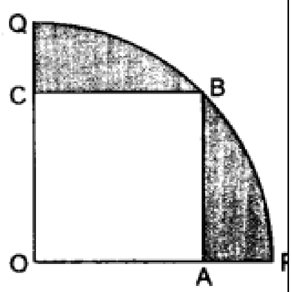
= 2−√ x (20) = 202−√cm

Radius of the quadrant of circle = Diagonal of square = 202−√

$$\text{Area of the quadrant OPBQ} = \frac{\pi r^2\theta}{360^\circ}$$

$$= \frac{3.14 \times 20\sqrt{2} \times 20\sqrt{2} \times 90^\circ}{360^\circ}$$

$$= 314 \times 2 = 628\ \text{cm}^2$$



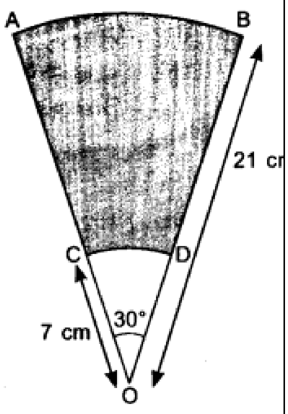
Question 14.

AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see figure). If ∠AOB=30°, find the area of the shaded region.

Solution:

Given: ∠AOB = 30°

Radius of the sector AOB = 21 cm



$\therefore$ Area of the sector AOB $= \frac{\pi r^2 \theta}{360°}$

$= \frac{22}{7} \times \frac{21 \times 21 \times 30°}{360°}$

$= \frac{11 \times 21}{2} = \frac{231}{2}$ cm^2

Radius of the sector COD = 7 cm

∠COD = 30°

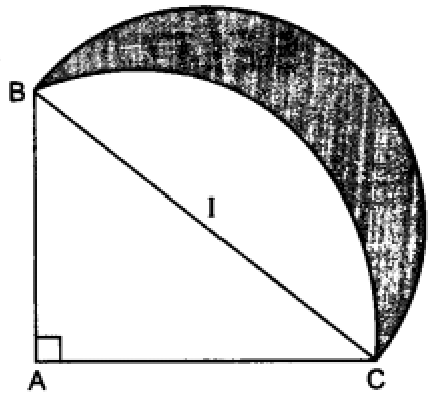
$\therefore$ Area of the sector COD $= \frac{\pi r^2 \theta}{360°} = \frac{22}{7} \times \frac{7 \times 7 \times 30°}{360°} = \frac{77}{6}$ cm^2

Area of the shaded region = Area of the sector AOB – Area of the sector COD

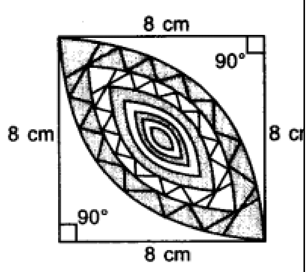
$= \frac{231}{2} - \frac{77}{6} = \frac{693 - 77}{6} = \frac{616}{6} = \frac{308}{3}$ cm^2

Question 15.

In the figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Solution:

Radius of the quadrant of the circle = 14 cm Area of the quadrant of the circle $= \frac{\pi r^2\theta}{360°}$ $= \frac{22}{7} \times \frac{14 \times 14 \times 90}{360}$ $= 11 \times 14 = 154\ cm^2$ Area of the $\Delta BAC = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 14 \times 14 = 7 \times 14 = 98\ cm^2$ Area of the region marked as I = Area of the quadrant – Area of the ΔABC $= 154 - 98 = 56\ cm^2$ Area of the semicircle on diameter BC $= \frac{1}{2} \times \pi r^2$ where, $r = \frac{1}{2} BC = \frac{1}{2}\sqrt{2}(AC)$ $\quad \left(\because BC^2 = AB^2 + AC^2 = AC^2 + AC^2,\ BC^2 = 2AC^2,\ BC = \sqrt{2} AC \right.$ $r = \frac{1}{2}(\sqrt{2} \times 14)$ $\Rightarrow \quad r = 7\sqrt{2}\ cm$
Area of the shaded region = Area of the semicircular region – Area of the region I $= 154\ cm^2 - 56\ cm^2 = 98\ cm^2$
Question 16. Calculate the area of the designed region in the figure common between the two quadrants of the circles of the radius 8 cm each. **Solution:** Side of the square = 8 cm Area of the square = 8 x 8 = 64 cm² Radius of the quadrant (formed at vertex) = 8 cm  Area of the quadrant $= \frac{\pi r^2\theta}{360°}$ $= \frac{22}{7} \times \frac{8 \times 8 \times 90°}{360°} = \frac{22 \times 2 \times 8}{7} = \frac{352}{7}$ Area of the square left on subtracting area of one quadrant = Area of the square – Area of the quadrant $= 64 - \frac{352}{7} = \frac{448 - 352}{7} = \frac{96}{7}\ cm^2$ Area of the shaded region = Area of the square – 2 × Area of the square left on subtracting area of one quadrant $= 64 - 2 \times \frac{96}{7} = \frac{448 - 192}{7} = \frac{256}{7}\ cm^2$

LBYH

LBYH