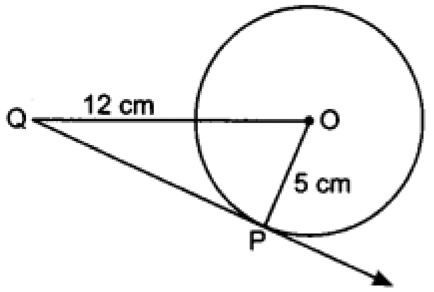
NCERT Solutions Of Chapter 10 - Circles

Ex 10.1

Question 1.
How many tangents can a circle have?
Solution:
There can be infinitely many tangents to a circle.

Question 2.
Fill in the blanks:
(i) A tangent to a circle intersects it in ………… point(s).
(ii) A line intersecting a circle in two points is called a ………… .
(iii) A circle can have parallel tangents at the most ………… .
(iv) The common point of a tangent to a circle and the circle is called ……….. .
Solution:
(i) One
(ii) Secant
(iii) Two
(iv) Point of contact.

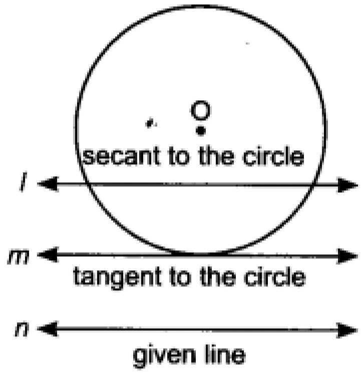
Question 3.
A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is
(a) 12 cm
(b) 13 cm
(c) 8.5 cm
(d) 199−−−√ cm
Solution:



Radius of the circle = 5 cm
OQ = 12 cm
∠OPQ = 90°
[The tangent to a circle is perpendicular to the radius through the point of contact]

$PQ^2 = OQ^2 - OP^2$ [By Pythagoras theorem]
$PQ^2 = 12^2 - 5^2 = 144 - 25 = 199$
PQ = $\sqrt{199}$ cm.
Hence correct option is (d).

Question 4.
Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.
Solution:

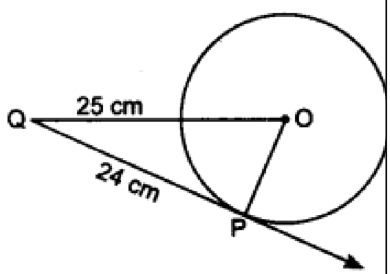


A line m is parallel to the line n and a line l which is secant is parallel to the given line.

Ex 10.2

Question 1.
From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is
(a) 7 cm Sol.
(b) 12 cm
(c) 15 cm
(d) 24.5 cm
Solution:

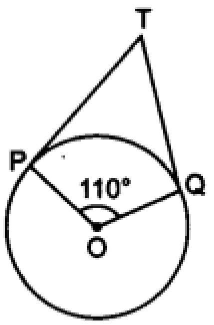
From figure $OQ^2 = OP^2 + PQ^2$
$(25)^2 = OP^2 + (24)^2$
$\Rightarrow 625 - 576 = OP^2$
$\Rightarrow 49 = OP^2 \Rightarrow OP = \sqrt{49}$
$\Rightarrow OP = 7$ cm
Radius of the circle = 7 cm
Hence, correct option is (*a*).



Question 2.
In figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ

= 110°, then ∠PTQ is equal to
(a) 60°
(b) 70°
(c) 80°
(d) 90°

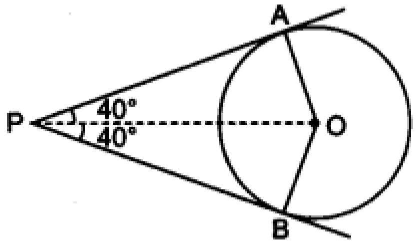
Solution:



∠OPT = 90°
∠OQT = 90°
∠POQ = 110°
TPOQ is a quadrilateral,
∴ ∠PTQ + ∠POQ = 180° ⇒ ∠PTQ + 110° = 180°
⇒∠PTQ = 180°- 110° = 70°
Hence, correct option is (b).

Question 3.
If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to
(a) 50°
(b) 60°
(c) 70°
(d) 80°

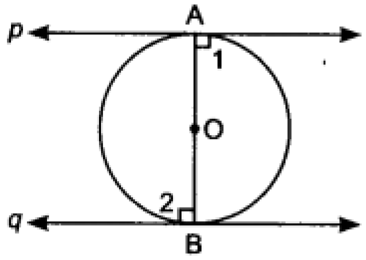
Solution:
In AOAP and AOBP
OA = OB [Radii]
PA = PB



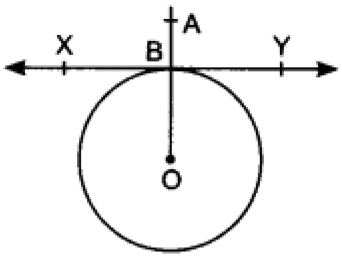
[Lengths of tangents from an external point are equal]

OP = OP [Common]
∴ ΔOAP ≅ ΔOBP [SSS congruence rule]
∠AOB + ∠APB = 180° ⇒ ∠AOB + 80° = 180°
⇒∠AOB = 180° – 80° = 100°
From eqn. (i), we get
⇒∠POA = 12 x 100° = 50°
Hence, correct option is (a)

Question 4.
Prove that the tangents drawn at the ends of a diameter of a circle are parallel.
Solution:
AB is a diameter of the circle, p and q are two tangents.
OA ⊥ p and OB ⊥ q
∠1 = ∠2 = 90°
⇒ p || q ∠1 and ∠2 are alternate angles]



Question 5.
Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.
Solution:
XY tangent to the circle C(0, r) at B and AB ⊥ XY. Join OB.
∠ABY = 90° [Given]
∠OBY = 90°
[Radius through point of contact is perpendicular to the tangent]
∴ ∠ABY + ∠OBY = 180° ⇒ ABOiscollinear
∴ AB passes through centre of the circle.



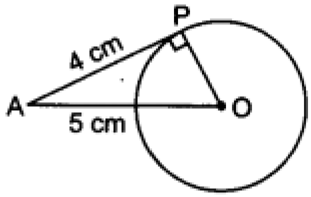
Question 6.
The length of a tangent from a point A at distance 5 cm from the centre of the

circle is 4 cm. Find the radius of the circle.

Solution:

OA = 5 cm, AP = 4 cm OP = Radius of the circle

∠OPA = 90° [Radius and tangent are perpendicular]



$$5^2 = 4^2 + OP^2 \Rightarrow 25 = 16 + OP^2$$

$$\Rightarrow 25 - 16 = OP^2 \Rightarrow 9 = OP^2 \Rightarrow \sqrt{9} = OP$$

$$\Rightarrow OP = 3 \text{ cm}$$

$$\therefore \text{Radius} = 3 \text{ cm}$$

Question 7.

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

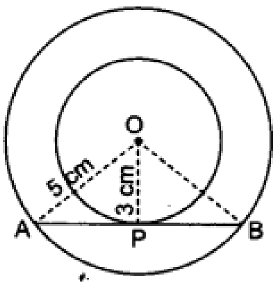
Solution:

Radius of larger circle = 5 cm Radius of smaller circle = 3 cm

OP ⊥ AB

[Radius of circle is perpendicular to the tangent]

AB is a chord of the larger circle



∴ OP bisects AB

$$\therefore AP = BP$$

In ΔOAP $OA^2 = AP^2 + OP^2 \Rightarrow (5)^2 = AP^2 + (3)^2$

$$\Rightarrow 25 - 9 = AP^2 \Rightarrow 16 = AP^2$$

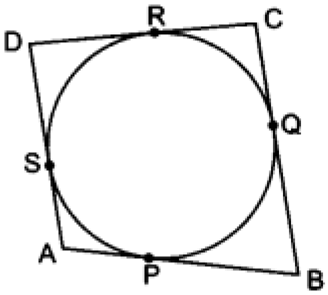
$$\Rightarrow \sqrt{16} = AP \Rightarrow AP = 4 \text{ cm}$$

Hence, length of the chord, $AB = 2AP = 2 \times 4 = 8$ cm

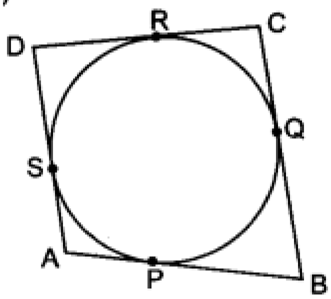
Question 8.

A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that AB + CD = AD + BC.

Solution:
AP = AS … (i)
[Lengths of tangents from an external point are equal]
BP = BQ … (ii)
CR = CQ … (iii)
DR = DS … (iv)

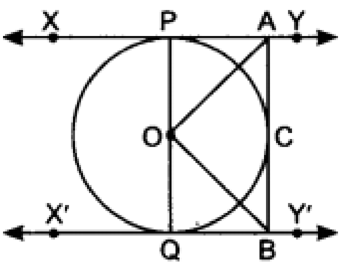


Adding equations (i), (ii), (iii) and (iv), we get



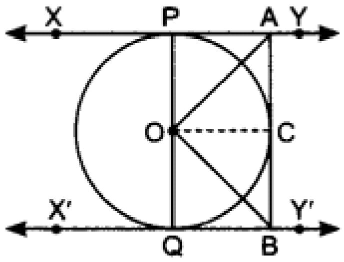
AP + BP + CR + DR = AS + BQ + CQ + DS
⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
⇒ AB + CD = AD + BC
Hence proved.

Question 9.
In figure, XY and X'Y' are two parallel tangents to a circle , x with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90°.



Solution:
Given: Two parallel tangents to a circle with centre O. Tangent AB with point of contact C intersects XY at A and X'Y' at B To Prove: ∠AOB = 90° with point of

contact C intersects XY at A and X'Y' at B



To Prove: ∠AOB = 90°
Construction: Join OA, OB and OC
Proof: In ΔAOP and ΔAOC

AP = AC [Lengths of tangents]
OP = OC [Radii]
OA = OA [Common]
⇒ ΔAOP ≅ ΔAOC [SSS congruence rule]
⇒ ∠PAO = ∠CAO [C.P.C.T]
∴ ∠PAC = 2 ∠OAC ...(i)
Similarly ∠QBC = 2 ∠OBC ...(ii)

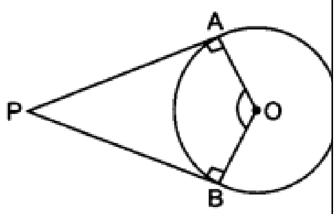
Adding (i) and (ii), we get
∠PAC + ∠QBC = 2[∠OAC + ∠OBC]
∵ ∠PAC + ∠QBC = 180°
[interior consecutive angle on same side of transversal]
∴ 180° = 2[∠OAC + ∠OBC]
⇒ ∠OAC + ∠OBC = 90°

In ΔAOB,
∠AOB + [∠OAC + ∠OBC] = 180° ⇒ ∠AOB + 90° = 180°
⇒ ∠AOB = 180° – 90° = 90°

Question 10.
Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.
Solution:

PA and PB are two tangents, A and B are the points of contact of the tangents.

$$OA \perp AP \text{ and } OB \perp BP$$



$$\angle OAP = \angle OBP = 90°$$

[Radius and tangent are perpendicular to each other]

In quadrilateral OAPB

$$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$$

$$\Rightarrow \quad 90° + 90° + \angle APB + \angle AOB = 360°$$

$$\angle APB + \angle AOB = 360° - 180° = 180°$$

Hence, $\angle APB$ and $\angle AOB$ are supplementary angles.

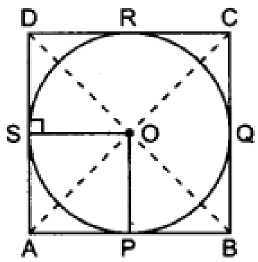
Question 11.

Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Parallelogram ABCD circumscribing a circle with centre O.

$OP \perp AB$ and $OS \perp AD$



In ΔOPB and ΔOSD

$$\angle OPB = \angle OSD \quad \text{[Each 90°]}$$

$$OB = OD$$

[Diagonals of a parallelogram bisect each other]

$$OP = OS \quad \text{[Radii]}$$

$$\Rightarrow \quad \Delta OPB \cong \Delta OSD \quad \text{[RHS congruence rule]}$$

$$PB = SD \quad ...(i) \quad \text{[C.P.C.T]}$$

$$AP = AS \quad ...(ii) \quad \text{[Lengths of tangent]}$$

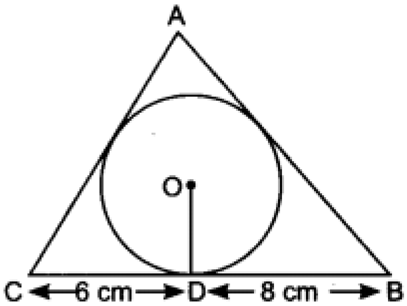
Adding (*i*) and (*ii*), $AP + PB = AS + DS$

$$AB = AD$$

Similarly $AB = BC = CD = DA$

Question 12.

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.

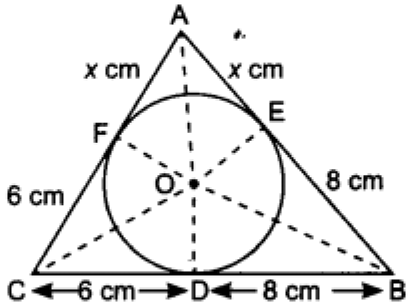


Solution:

BD = 8 cm and DC = 6 cm

BE = BD = 8 cm

CD = CF = 6 cm



Let AE = AF = x cm

In ΔABC, a = 6 + 8 = 14 cm

b = (x + 6) cm

c = (x + 8) cm

$$s = \frac{a+b+c}{2} = \frac{14+x+6+x+8}{2} = \frac{2x+28}{2} = (x + 14) \text{ cm}$$

$$\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(x+14)\times x\times 8\times 6} = \sqrt{48x\times(x+14)} \text{ cm}^2 \quad ...(i)$$

Again, $\text{ar}(\Delta ABC) = \text{ar}(\Delta OBC) + \text{ar}(\Delta OCA) + \text{ar}(\Delta OAB)$

$$= \frac{1}{2}\times 4\times a + \frac{1}{2}\times 4\times b + \frac{1}{2}\times 4\times c$$

$$= 2a + 2b + 2c = 2(a + b + c) = 2 \times 2(x + 14) \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$\sqrt{48x(x+14)} = 4(x+14) \Rightarrow 48x(x + 14) = 4^2(x + 14)^2$$

$$\Rightarrow 48x(x + 14) = 16(x + 14)^2 \Rightarrow 3x(x + 14) = (x + 14)^2$$

$$\Rightarrow 3x = x + 14 \Rightarrow 2x = 14 \Rightarrow x = 7$$

$$AB = x + 8 = 7 + 8 = 15 \text{ cm}$$

$$AC = x + 6 = 7 + 6 = 13 \text{ cm}$$

Question 13.

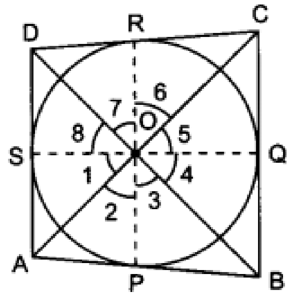
Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

AB touches at P.

BC, CD and DA touch the circle at Q, R and S.

Construction: Join OA, OB, OC, OD and OP, OQ, OR, OS.



OA bisects ∠POS

$\therefore \quad \angle 1 = \angle 2$

Similarly $\quad \angle 3 = \angle 4$

$\angle 5 = \angle 6$

$\angle 7 = \angle 8$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360°$

$2[\angle 1 + \angle 4 + \angle 5 + \angle 8] = 360°$

$(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 180°$

$\angle AOD + \angle BOC = 180°$

Similarly, $\quad \angle AOB + \angle COD = 180°$

Hence, opposite sides of quadrilateral circumscribing a circle subtend supplementary angles at the centre of a circle.