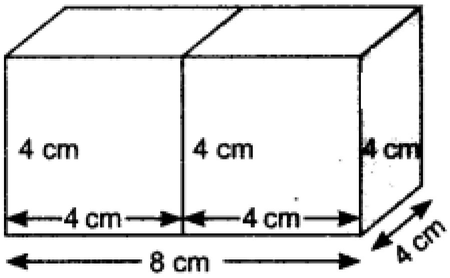
NCERT Solution Of Chapter 13 – Surface Area And Volumes

Ex 13.1

Unless stated otherwise, take π = 227

13.1 Class 10 Question 1.

2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

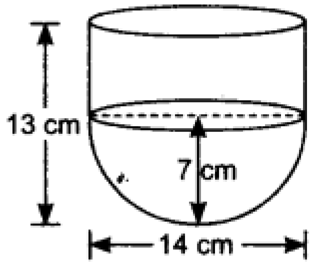


Solution:

Volume of one cube = 64 cm^3
Let edge of one cube = a
Volume of the cube = $(edge)^3$
$a^3 = 64 \Rightarrow a = 4$ cm
Similarly, edge of the another cube = 4 cm.
Now, both cubes are joined together and a cuboid is formed as shown in the figure.
Now, length of the cuboid (l) = 8 cm
breadth of the cuboid (b) = 4 cm
height of the cuboid (h) = 4 cm
Surface area of the cuboid so formed = 2 (lb + bh + hl)
= 2(8 x 4 + 4 x 4 + 4 x 8)
= 2(32 + 16 + 32) = 160 cm^2

NCERT Class 10 Exercise 13.1 Question 2.

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

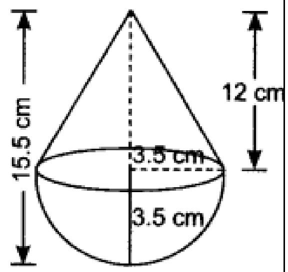


Solution:
Given: diameter of the hemisphere = 14 cm
Radius = $\frac{14}{2}$ = 7 cm
Curved surface area of the hemisphere = $2\pi r^2$ = 2 x $\frac{22}{7}$ x 7 x 7 cm^2
= 14 x 22 cm^2 = 308 cm^2
Here, total height of the vessel = 13 cm
Height of the cylinder = Total height – Height of the hemisphere = 13 cm – 7 cm = 6 cm
and radius of the cylinder = radius of the hemisphere = 7 cm
Inner surface area of the cylinder = $2\pi rh$ = 2 x $\frac{22}{7}$ x 7 x 6
= 2 x 22 x 6 = 264 cm^2
Inner surface area of the vessel = Inner surface area of the cylinder + curved surface area of the hemisphere
= 264 cm^2 + 308 cm^2 = 572 cm^2

Ex 13.1 Class 10 Question 3.
A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Solution:

Given: radius of the hemisphere $= 3.5$ cm



Surface area of the hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 3.5 \times 3.5 \text{ cm}^2$$

$$= \frac{2 \times 22 \times 35 \times 35}{7 \times 10 \times 10} = 77 \text{ cm}^2$$

Height of the conical portion $= 15.5 \text{ cm} - 3.5 \text{ cm} = 12$ cm

Radius of the conical portion $= 3.5$ cm

Slant height of the conical portion $= \sqrt{(12)^2 + (3.5)^2}$ cm

($\because l^2 = r^2 + h^2$, l = slant height, r = radius, h = height)

$$= \sqrt{144 + 12.25} \text{ cm} = \sqrt{156.25} = 12.5 \text{ cm}$$

Curved surface area of the conical portion $= \pi r l = \pi \times 3.5 \times 12.5$

$$= \frac{22}{7} \times \frac{35}{10} \times \frac{125}{10} \text{ cm}^2 = \frac{11 \times 25}{2} \text{ cm}^2 = \frac{275}{2} \text{ cm}^2$$

Total surface area of the toy = Surface area of the hemisphere + Surface area of the conical portion

$$= 77 \text{ cm}^2 + \frac{275}{2} \text{ cm}^2 = \frac{154 + 275}{2} \text{ cm}^2$$

$$= \frac{429}{2} \text{ cm}^2 = 214.5 \text{ cm}^2$$

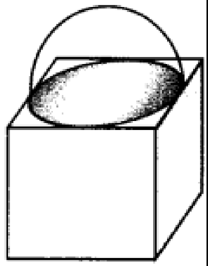
Surface Area And Volume Class 10 Question 4.

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Solution:

Given: edge of the cube = 7 cm

The greatest diameter the hemisphere can have = edge of the cube = 7 cm



Radius of the hemisphere $= \frac{7}{2}$ cm

Surface area of the hemisphere $= 2\pi r^2$

$$= \frac{2 \times 22 \times 7 \times 7}{7 \times 2 \times 2} = 77 \text{ cm}^2$$

Area of circular portion covered by the hemisphere $= \pi r^2$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ cm}^2$$

Total surface area of the cubical block

$$= 6(7)^2 = 6 \times 7 \times 7 = 294 \text{ cm}^2$$

Total surface area of the solid = (Surface area of the cubical portion + surface area of the hemisphere – area of the circular region)

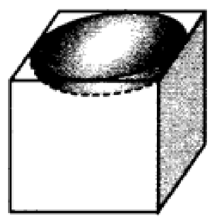
$$= 294 \text{ cm}^2 + 77 \text{ cm}^2 - \frac{77}{2} \text{ cm}^2 = 294 \text{ cm}^2 + \frac{77}{2} \text{ cm}^2$$

$$= 294 \text{ cm}^2 + 38.5 \text{ cm}^2 = 332.5 \text{ cm}^2$$

Class 10 Maths Chapter 13 Question 5.

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution:



Given: edge of the cube $= l$

Surface area of the cube $= 6(\text{edge})^2 = 6l^2$

Diameter of the hemisphere $= l$

$\therefore$ Radius of the hemisphere $= \frac{l}{2}$

Inner surface area of the hemisphere $= 2\pi r^2 = 2\pi\left(\frac{l}{2}\right)^2 = \frac{2\pi l^2}{4} = \frac{\pi l^2}{2}$

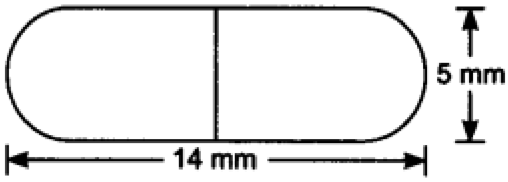
Area of circular portion of the hemisphere $= \pi r^2 = \pi\left(\frac{l}{2}\right)^2 = \frac{\pi l^2}{4}$

Remaining surface area of the cubical box = (Surface area of the cubical box + Inner surface area of the hemisphere – Area of the circular region)

$$= 6l^2 + \frac{\pi l^2}{2} - \frac{\pi l^2}{4} = 6l^2 + \frac{\pi l^2}{4} = \frac{l^2}{4}(24 + \pi)$$

Chapter 13 Class 10 Maths Question 6.

A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Solution:

Diameter of each hemispherical ends = 5 mm

$$\text{Radius} = \frac{5}{2} \text{ mm}$$

Surface area of each hemispherical end $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} = \frac{275}{7} \text{ mm}^2$$

Surface area of both hemispherical ends $= \frac{2 \times 275}{7} \text{ mm}^2 = \frac{550}{7} \text{ mm}^2$

Total length of the capsule = 14 mm

Length of the cylindrical surface = Total length – radius of both hemispherical ends

$$= 14 \text{ mm} - 2\left(\frac{5}{2}\right) \text{ mm} = 14 - 5 = 9 \text{ mm}$$

Curved surface area of the cylindrical portion $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \times 9 \text{ mm}^2 = \frac{990}{7} \text{ mm}^2$$

Total surface area of the capsule = Area of both hemispherical ends + area of the cylindrical portion

$$= \frac{550}{7} \text{ mm}^2 + \frac{990}{7} \text{ mm}^2 = \frac{1540}{7} \text{ mm}^2 = 220 \text{ mm}^2$$

Class 10 Maths Chapter 13 Question 7.

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

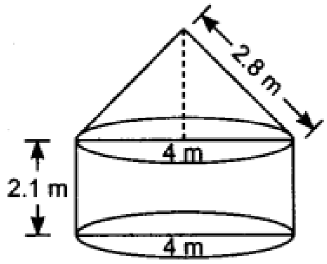
Solution:

Radius of the cylindrical part $= \frac{4}{2} = 2$ m

Height of the cylindrical part = 2.1 m

Curved surface area of the cylindrical part $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 2 \times 2.1 \text{ m}^2$$
$$= 4 \times 22 \times 0.3 \text{ m}^2$$
$$= 22 \times 1.2 = 26.4 \text{ m}^2$$



Curved surface area of the top $= \pi rl$

$$= \frac{22}{7} \times 2 \times 2.8 \text{ m}^2 = 44 \times 0.4 \text{ m}^2 = 17.6 \text{ m}^2$$

Total area of the canvas $= 26.4 + 17.6 = 44 \text{ m}^2$

Cost of the canvas $=$ ₹ $500/\text{m}^2$

Total cost $=$ Cost of canvas per $\text{m}^2 \times$ Total surface area of the canvas

Total cost $= 44 \times 500 =$ ₹ 22,000

Exercise 13.1 Class 10 NCERT Solutions Question 8.

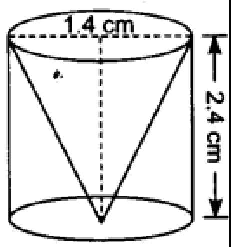
From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

Solution:

Diameter of base = 1.4 cm

$$\text{Radius of base} = \frac{1.4}{2} = 0.7 \text{ cm}$$

Slant height of the conical cavity



$$l = \sqrt{\begin{pmatrix}\text{Radius of}\\ \text{base}\end{pmatrix}^2 + \begin{pmatrix}\text{Height of}\\ \text{cylinder}\end{pmatrix}^2}$$

$$l = \sqrt{(0.7)^2 + (2.4)^2}$$

$$= \sqrt{0.49 + 5.76}$$

$$l = \sqrt{6.25} = 2.5 \text{ cm}$$

Inner surface area of the conical cavity $= \pi r l$

$$= \frac{22}{7} \times 0.7 \times 2.5 \text{ cm}^2 = \frac{22 \times 25}{100} \text{ cm}^2 = \frac{11}{2} \text{ cm}^2 = 5.5 \text{ cm}^2$$

Curved surface area of the cylinder $= 2\pi r h$

$$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 \text{ cm}^2 = 2 \times 22 \times \frac{1}{10} \times \frac{24}{10} \text{ cm}^2$$

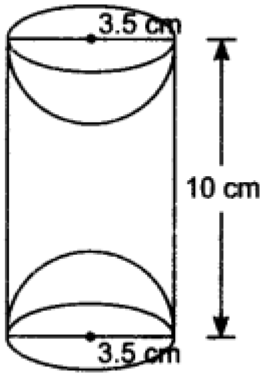
$$= \frac{44 \times 24}{100} \text{ cm}^2 = \frac{1056}{100} \text{ cm}^2 = 10.56 \text{ cm}^2$$

$$\text{Area of base of the cylinder} = \pi r^2 = \frac{22}{7} \times 0.7 \times 0.7 \text{ cm}^2 = \frac{22 \times 7}{100} \text{ cm}^2 = 1.54 \text{ cm}^2$$

Total surface area of the remaining solid = (5.5 + 10.56 + 1.54) cm^2 = 17.60 cm^2

Class 10 Maths Chapter 13 Solutions Question 9.

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.



Solution:

Given: height of the cylinder = 10 cm,

base radius = 3.5 cm

Curved surface area of the cylinder $= 2\pi rh$

$$= \frac{2 \times 22 \times 35 \times 10}{7 \times 10} \text{ cm}^2 = 220 \text{ cm}^2$$

Inner surface area of a hemispherical cavity

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times \frac{35 \times 35}{10 \times 10} \text{ cm}^2 = 77 \text{ cm}^2$$

Inner surface area of both hemispherical cavity $= 77 \text{ cm}^2 + 77 \text{ cm}^2 = 154 \text{ cm}^2$

Total surface area of the solid = Curved surface area of the solid + Inner surface area of both hemispherical ends

$$= 220 \text{ cm}^2 + 154 \text{ cm}^2 = 374 \text{ cm}^2$$

Ex 13.2

Unless stated otherwise, take π = 227

Question 1.

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of n.

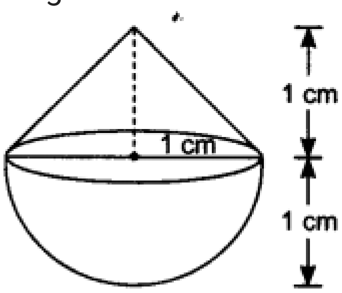
Solution:

Radius of the hemisphere = 1 cm

Volume of the hemisphere = $23\pi r^3 = 23\pi(1)^3 = 23\pi cm^3$

Radius of base of the cone = 1 cm

Height of the cone = 1 cm



Volume of the cone = $13\pi r^2h = 13\pi r^2 \times 1 = 13\pi \text{ cm}^3$

Total volume of the solid = Volume of the hemisphere + Volume of the cone

= $23\pi \text{ cm}^3 + 13\pi \text{ cm}^3 = \pi \text{ cm}^3$

Question 2.

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of

2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Solution:

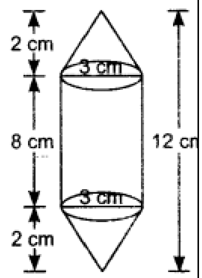
Volume of air contained in the model = Total volume of the solid

Diameter of base of each cone = 3 cm

∴ Radius of base of each cone = 32

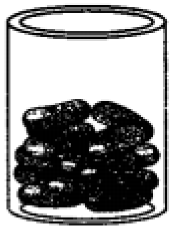
Height of each cone = 2 cm

$$\text{Volume of one cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times 2\ \text{cm}^3$$
$$= \frac{1}{3}\pi\left(\frac{9\times 2}{4}\right) = \frac{3}{2}\pi\ \text{cm}^3$$
$$\therefore \text{Volume of both cones} = 2\times\frac{3}{2}\pi\ \text{cm}^3 = 3\pi\ \text{cm}^3$$
$$\text{Volume of the cylindrical portion} = \pi r^2 h = \pi\left(\frac{3}{2}\right)^2 \times 8\ \text{cm}^3 = \frac{\pi\times 9\times 8}{4}\ \text{cm}^3 = 18\pi\ \text{cm}^3$$
$$\text{Volume of air contained in the model} = \text{Total volume of the solid}$$
$$= 3\pi\ \text{cm}^3 + 18\pi\ \text{cm}^3 = 21\pi\ \text{cm}^3$$
$$= \frac{21\times 22}{7}\ \text{cm}^3 = 66\ \text{cm}^3$$



Question 3.

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see figure).



Solution:

Volume of one piece of gulab jamun

= Volume of the cylindrical portion + Volume of the two hemispherical ends 1 2 8

Radius of each hemispherical portion = 2.82 = 1.4 cm

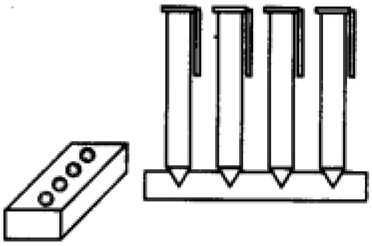
$$\text{Volume of one hemispherical ends} = \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}(1.4)^3\ \text{cm}^3$$
$$= \frac{2}{3}\times\frac{22}{7}\times 1.4\times 1.4\times 1.4\ \text{cm}^3$$
$$= \frac{2\times 22\times 2\times 14\times 14}{3\times 10\times 10\times 10}\ \text{cm}^3 = 5.74\ \text{cm}^3$$

Volume of both hemispherical ends = 2 x 5.74 cm³ = 11.48 cm³

Height of the cylindrical portion = (total height) – (radius of both hemispherical

ends)
= 5 cm – 2(1.4) cm = 5 cm – 2.8 cm = 2.2 cm
Radius of the cylindrical portion = 1.4 cm
Volume of the cylindrical portion of gulab jamun = $\pi r^2 h$
= 227 x $(1.4)^2$ x 2.2cm^3
= 22×1.4×1.4×2.27cm^3 = 13.55 cm^3
Total volume of one gulab jamun = Volume of the two hemispherical ends + Volume of the cylindrical portion
= 11.48 cm^3 + 13.55 cm^3 = 25.03 cm^3
Volume of sugar syrup = 30% of volume of gulab jamun
= 30100 x 25.03 cm^3 = 7.50 cm^3
∴ Volume of sugar syrup in 45 gulab jamuns
= 45 (volume of sugar syrup in one gulab jamun)
= 45 x 7.50 cm^3 = 337.5 cm^3 = 338 cm^3 approx.

Question 4.
A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see figure).



Solution:
Radius of one conical depression = 0.5 cm
Depth of one conical depression = 1.4 cm

$$\text{Volume of one conical depression} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4\,\text{cm}^3$$
$$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4\,\text{cm}^3 = 0.366\,\text{cm}^3$$

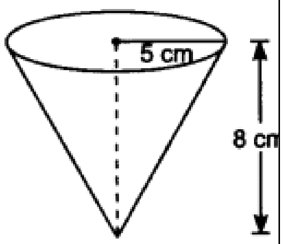
∴ Volume of four conical depressions
$$= 4 \times 0.366\,\text{cm}^3 = 1.464\,\text{cm}^3$$

Volume of cuboidal box = l x b x h
= 15 x 10 x 3.5 cm^3
= 525 cm^3
Remaining volume of box = Volume of cubical box – Volume of four conical

depressions
= 525 cm³ – 1.464 cm³ = 523.5 cm³

Question 5.
A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.
Solution:

Given: height of the cone = 8 cm
and radius of the cone = 5 cm



$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(5)^2 8 \text{ cm}^3 = \frac{200}{3}\pi \text{ cm}^3$$

Radius of one spherical lead shot = 0.5 cm

$\therefore$ Volume of one spherical lead shot $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.5)^3 \text{ cm}^3$

$$= \frac{4 \times 0.125}{3}\pi \text{ cm}^3 = \frac{0.5}{3}\pi \text{ cm}^3$$

When spherical lead are dropped in the vessel, one fourth of water flows out

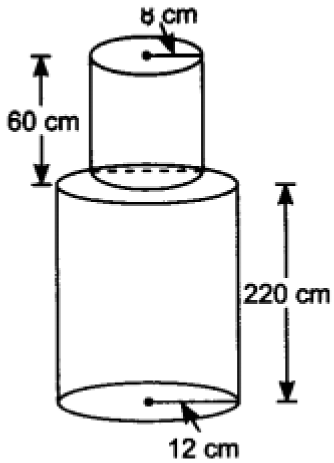
Let number of lead shots = n

Volume of n spherical shots $= \frac{1}{4}$ volume of conical vessel

$$\Rightarrow \quad n\left(\frac{0.5}{3}\pi\right) = \frac{1}{4}\left(\frac{200}{3}\pi\right) \Rightarrow n(0.5) = 50$$

$$\Rightarrow \quad n = \frac{50 \times 10}{5} = 100$$

Question 6.
A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm3 of iron has approximately 8 g mass. (Use π = 3.14)



Solution:
Given: radius of 1st cylinder = 12 cm
and height of 1st cylinder = 220 cm
∴ Volume of 1st cylinder = $\pi r^2 h$
= $\pi(12)^2 (220)$ cm³
= 144 x 220π cm³
= 144 x 220 x 3.14 cm³
= 99475.2 cm³ … (i)
Given: radius of 2nd cylinder = 8 cm
and height of 2nd cylinder = 60 cm
∴ Volume of 2nd cylinder = $\pi r^2 h$
= $\pi(8)^2 (60)$ cm³ = 64 x 60π cm³
= 64 x 60 x 3.14 cm³
= 12057.6 cm³ … (ii)
Total volume of solid = Volume of 1st cylinder + Volume of 2nd cylinder
= 99475.2 cm³ + 12057.6 cm³ = 111532.8 cm³
Given: mass of 1 cm³ of iron = 8 g
∴ Mass of 111532.8 cm³ of iron = 111532.8 x 8 g
= 892262.4 g = 892.262 kg

Question 7.
A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.
Solution:
Radius of hemisphere = 60 cm

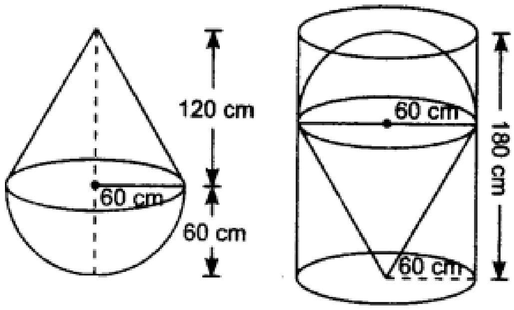
Volume of hemisphere $= \frac{2}{3}\pi(60)^3 \text{ cm}^3$

Radius of conical base $= 60$ cm

Height of conical portion $= 120$ cm

Volume of conical portion $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(60)^2 \times 120 \text{ cm}^3$



Total volume of hemispherical and conical solid = Volume of hemisphere + Volume of conical portion

$= \frac{2}{3}\pi (60)^3 \text{ cm}^3 + \frac{1}{3}\pi(60)^2\, 120 \text{ cm}^3$

$= \frac{1}{3}\pi(60)^2 [2(60) + 120] \text{ cm}^3$

$= \frac{1}{3}\pi \times 60 \times 60 \times 240 \text{ cm}^3 = \pi(60)^2 \times 80 \text{ cm}^3$

Volume of cylindrical solid $= \pi r^2 h = \pi(60)^2 \times 180 \text{ cm}^3$

[where $r = 60$ cm and $h = 180$ cm]

When solid (hemisphere + conical) is kept in cylindrical solid, then volume of water left in cylinder

= Volume of cylinder – (Volume of hemisphere + Volume of cone)

$= [\pi(60)^2 \times 180 - \pi(60)^2 \times 80] \text{ cm}^3$

$= \pi(60)^2 [180 - 80] \text{cm}^3 = \pi \times 3600 \times 100 \text{ cm}^3 = 1130400 \text{ cm}^3 = 1.130 \text{ m}^3$

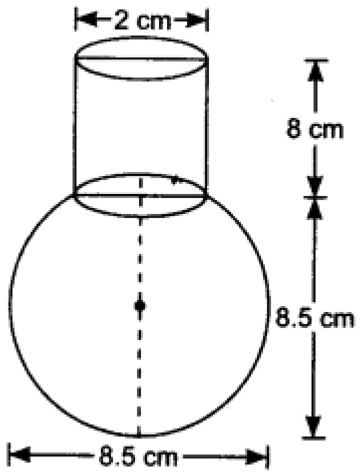
Question 8.

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter, the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Solution:

Volume of water the glass vessel can hold = 345 cm^3 (Measured by the child)

Radius of the cylindrical part = 22 = 1 cm



Height of the cylindrical part = 8 cm
∴ Volume of the cylindrical part = $\pi r^2 h$
= 3.14 x $(1)^2$ x 8 cm^3
Diameter of the spherical part Radius = 8.5 cm
∴ Radius = 8.52 cm = 8520

$$\therefore \quad \text{Volume of the spherical part} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times \frac{85 \times 85 \times 85}{20 \times 20 \times 20} = 321.39 \text{ cm}^3$$

Total volume of the glass vessel = Volume of the cylindrical part + Volume of the spherical part
= 25.12 cm^3 + 321.39 cm^3 = 346.51 cm^3
Volume measured by child is 345 cm^3, which is not correct. Correct volume is 346.51 cm^3.

Ex 13.3

Unless stated otherwise, take π = 227

Question 1.
A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.
Solution:
Given: radius of metallic sphere = 4.2 cm
∴ Volume = 43π$(4.2)^3$ …. (i)
∵ Sphere is melted and recast into a cylinder of radius 6 cm and height h.
∴ Volume of the cylinder =$\pi r^2 h$ = $\pi(6)^2$ x h … (ii)
According to question,

Volume of the cylinder = Volume of the sphere

$$\Rightarrow \quad \pi(6)^2 h = \frac{4}{3}\pi(4.2)^3 \quad \Rightarrow \quad 36h = \frac{4}{3} \times \frac{42 \times 42 \times 42}{1000}$$

$$\Rightarrow \quad h = \frac{4 \times 42 \times 42 \times 42}{36 \times 3 \times 1000} \text{ cm} \quad \Rightarrow \quad h = \frac{4 \times 7 \times 7 \times 14}{1000} \text{ cm}$$

$$\Rightarrow \quad h = \frac{2744}{1000} \text{ cm} = 2.74 \text{cm}$$

Question 2.
Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.
Solution:
Radius of 1st metallic sphere = 6 cm
∴ Volume of 1^{st} metallic sphere = $43\pi(6)^3$ cm^3
Radius of 2^{nd} metallic sphere = 8 cm
∴ Volume of 2^{nd} metallic sphere = $43\pi(8)^3$ cm^3
Radius of 3^{rd} metallic sphere = 10 cm
∴ Volume of 3^{rd} metallic sphere = $43\pi(10)^3$ cm^3
Volume of all three metallic spheres = $43\pi(6^3+8^3+10^3)$ cm^3
∵ 3 spheres are melted and recast into a new metallic sphere of radius r.
∴ Volume of new metallic sphere = $43\pi r^3$

According to question,

$$\frac{4}{3}\pi(6^3+8^3+10^3) = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad 6^3+8^3+10^3 = r^3$$

$$\Rightarrow \quad 216+512+1000 = r^3 \quad \Rightarrow \quad 1728 = r^3 \quad \Rightarrow \quad r = 12 \text{ cm}$$

Question 3.
A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.
Solution:
Given: diameter of the well = 7 m Radius = 72m
and depth of the well = 20 m

Volume of the earth taken out from the well = πr²

$$= \pi\left(\frac{7}{2}\right)^2 \times 20\ \text{m}^3 = \pi \times \frac{49}{4} \times 20 = 49 \times 5 \times \pi$$

Earth is evenly spread on the platform 22 m by 14 m

Let height of the platform = h m

∴ Volume of the platform = $22 \times 14 \times h$

According to question,

Volume of the platform = Volume of the earth taken out from the well

$$22 \times 14 \times h = \pi \times 49 \times 5$$

$$\Rightarrow \quad 22 \times 14 \times h = \frac{22}{7} \times 49 \times 5$$

$$\Rightarrow \quad h = \frac{49 \times 5 \times 22}{7 \times 22 \times 14}$$

$$\Rightarrow \quad h = \frac{5}{2} = 2.5\ \text{m}$$

Question 4.

A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Solution:

Given: diameter of the well = 3 m

⇒ Radius = 32m

Depth of the well = 14 m

Volume of the earth taken out from the well = πr²h

= π(32)² x 14 = π×9×144 = 632πm³

∵ Earth taken out from the well evenly spread to form an embankment having height h and width of embankment around the well is 4 m.

∴ External radius (R) = radius of well + width of the embankment

= 32m + 4m = 112m

Internal radius = 32m = radius of well

Volume of the earth used for embankment = $\pi(R^2 - r^2)h$

$$= \pi\left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]h\ \text{m}^3 = \pi\left(\frac{121}{4} - \frac{9}{4}\right)h\ \text{m}^3$$

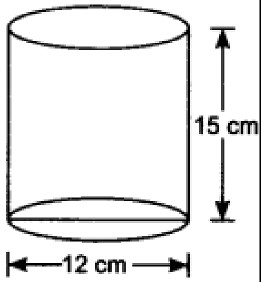
$$= \pi\left(\frac{112}{4}\right)h\ \text{m}^3 = \pi(28)h\ \text{m}^3$$

According to question,

$$\frac{63}{2}\pi = \pi \times 28h \quad \Rightarrow \quad h = \frac{63}{2 \times 28} = \frac{9}{8} = 1.125\ \text{m}$$

Question 5.
A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.
Solution:

Given: radius of cylinder $= \frac{12}{2}$ cm $= 6$ cm,



and height of cylinder $= 15$ cm

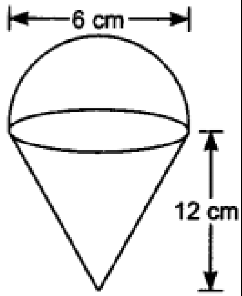
$\therefore$ Volume of cylinder $= \pi r^2 h = \pi(6)^2 \times 15 \text{ cm}^3$

$= \pi(36) \times 15 \text{ cm}^3$

Given: radius of ice cream cone $= \frac{6}{2} = 3$ cm

and height of ice cream cone $= 12$ cm

$\therefore$ Volume of ice cream cone $= \frac{1}{3}\pi(3)^2 \times 12 \text{ cm}^3$



Radius of hemispherical portion $= \frac{6}{2} = 3$ cm

$\therefore$ Volume of hemispherical portion $= \frac{2}{3}\pi(3)^3 \text{ cm}^3$

Total volume of ice cream in conical portion and hemisphere

$$= \frac{1}{3}\pi(9)(12) \text{ cm}^3 + \frac{2}{3}\pi(3)^3 \text{ cm}^3$$

$$= \pi\left[\frac{1}{3} \times 9 \times 12 + \frac{2}{3} \times 27\right] \text{ cm}^3$$

$$= \pi[36 + 18] = \pi[54] \text{ cm}^3$$

Let total number of ice cream cones are n.
∴ All ice cream cones are filled from ice cream in the cylinder.
Total volume of n number of ice cream cones = Volume of ice cream in the cylinder
n x π x 54 = π(36)15
⇒ n x 54 = 36 x 15
⇒ n = $\frac{36 \times 15}{54}$ = 10

Question 6.
How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm x 10 cm x 3.5 cm?
Solution:
Given: diameter of each coin = 1.75 cm ⇒ radius = $\frac{1.75}{2}$cm
and thickness of each coin = 2 mm

Let n number of coins are melted to form a cuboid.

$\therefore$ Total volume of coins used to form a cylinder $= n(\pi r^2 h) = n\left\{\pi\left(\frac{175}{2\times 100}\right)^2 \times \frac{2}{10}\right\}$ cm^3

Volume of cuboid $= l \times b \times h = 5.5 \times 10 \times 3.5$ cm^3

According to question,

Volume of cuboid = Volume of 'n' number of coins.

$$\Rightarrow \quad 5.5 \times 10 \times 3.5 = n\left\{\frac{22}{7} \times \left(\frac{7}{4\times 2}\right)^2 \times \frac{1}{5}\right\}$$

$$\Rightarrow \quad \frac{55 \times 10 \times 35 \times 7 \times 5 \times 16 \times 4}{22 \times 7 \times 7 \times 100} = n \quad \Rightarrow \quad n = 400$$

Question 7.

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Solution:

Given: radius of the cylindrical bucket = 18 cm

and height = 32 cm

∴ Volume of the cylindrical bucket = $\pi^2 h = \pi(18)^2 \times 32$ cm^3

Let radius of the conical heap = r cm

Given: height of the conical heap = 24 cm

∴ Volume of the conical heap = $13\pi(r^2)$ 24cm^3

According to question,

Volume of the cylinderical bucket = Volume of the conical heap

$$\Rightarrow \quad \pi(18)^2 \times 32 = \frac{1}{3}\pi(r^2)24 \quad \Rightarrow \quad (18)^2 32 = \frac{1}{3} \times r^2 \times 24$$

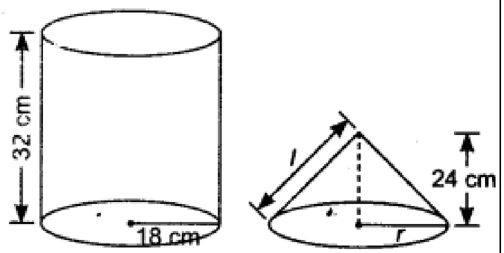
$$\Rightarrow \quad \frac{18 \times 18 \times 32 \times 3}{24} = r^2$$

$$\Rightarrow \quad 18 \times 6 \times 4 \times 3 = r^2$$

$$\Rightarrow \quad 6 \times 3 \times 6 \times 4 \times 3 = r^2$$

$$\Rightarrow \quad r = 6 \times 3 \times 2$$

$$\Rightarrow \quad r = 36 \text{ cm}$$



Let slant height of the cone = l cm

Now, consider right angled triangle in the cone.

$$l^2 = r^2 + h^2 \quad \text{(By pythagoras theorem)}$$

$$\Rightarrow \quad l^2 = (36)^2 + (24)^2 \quad \Rightarrow \quad l^2 = 1296 + 576 = 1872$$

$$\Rightarrow \quad l^2 = 144 \times 13 \quad \Rightarrow \quad l = 12\sqrt{13} \text{ cm}$$

Question 8.

Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Solution:

Given: width of canal = 6m, depth = 1.5 m
Rate of flowing water – 10 km/h
Volume of the water flowing in 30 minutes = 6×1.5×30×1060km³
= 6×1.5×10×1000×3010×60km³ = 45000 m³
We require water for standing up to height = 8 cm = 8100 m
Let the required area he A
∴ Volume of water required = A(8100)m³
According to question. 45000 = A×8100
⇒ 45000×1008 = A ⇒ A = 562500 m²

Question 9.
A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his Held, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?
Solution:
Given: diameter of the pipe = 20 cm ⇒ radius of the pipe = 10 cm
Water flowing from the pipe at rate = 3 km
Let it filled the tank in 't' hours.
Volume of the water flowing in 't' hours.

$$= \pi\left(\frac{10}{100}\right)^2 \times \frac{3 \times 1000 \times t}{1} \text{ m}^3 = \pi\left(\frac{3000}{100}\right)t \text{ m}^3 = \pi(30)t \text{ m}^3$$

Volume of the water in the cylindrical tank $= \pi r^2 h = \pi(5)^2 \times 2 \text{ m}^3$ $\left[r = \frac{10}{2} = 5 \text{ m}, h = 2 \text{ m}\right.$
According to question,

$$\pi(30)t = \pi(25) \times 2$$

$$\Rightarrow t = \frac{25 \times 2}{30} = \frac{10}{6} = \frac{5}{3} \text{ hours} = \frac{5}{3} \times 60 \text{ minutes} = 100 \text{ minutes.}$$

Ex 13.4

Unless stated otherwise, take π = 227

Question 1.
A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular .ends are 4 cm and 2 cm. Find the capacity of the glass.
Solution:
Given: upper diameter = 4 cm ⇒ upper radius = 12 = 2 cm = R
lower diameter = 2 cm ⇒ lower radius = 22 = 1 cm = r

height of glass = 14 cm

$$\text{capacity of the glass} = \frac{\pi h}{3}[R^2 + r^2 + Rr] = \frac{22}{7} \times \frac{14}{3}[2^2 + 1^2 + 2 \times 1]\ cm^3$$

$$= \frac{22}{7} \times \frac{14}{3}[4+1+2]cm^3 = \frac{22 \times 14 \times 7}{7 \times 3}cm^3 = \frac{22 \times 14}{3}cm^3 = 102.66cm$$

Question 2.
The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.
Solution:
Given: upper circumference of the frustum = 18 cm

$$\Rightarrow \quad 2\pi r_1 = 18\text{ cm} \Rightarrow \pi r_1 = \frac{18}{2} = 9\text{ cm} \quad ...(i)$$

and lower circumference,

$$2\pi r_2 = 6\text{ cm} \Rightarrow \pi r_2 = \frac{6}{2} = 3\text{ cm} \Rightarrow \pi r_2 = 3\text{ cm} \quad ...(ii)$$

Slant height $(l) = 4$ cm

Slant height (l) = 4 cm
We have C.S.A of the frustum = $\pi (r_1 + r_2)l$
Putting values from equation (i) and (ii), we get
Curved surface area = $(\pi r_1 + \pi r_2)l = (9 + 3) \times 4 = 12 \times 4 = 48\ cm^2$

Question 3.
A fez, the cap used by the Turks, is shaped like the frustum of a cone (see figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Solution:
Radius of open side (r_1) = 10 cm
Radius of upper base (r_2) = 4 cm
Slant height (l) = 15 cm

Curved surface area of the frustum $= \pi(r_1 + r_2)l = \pi(10+4)15 \text{ cm}^2$ $= \frac{22}{7} \times 14 \times 15 \text{ cm}^2 = 22 \times 2 \times 15 \text{ cm}^2 = 660 \text{ cm}^2$ Surface area of upper base $= \pi r_2^2 = \frac{22}{7} \times 4 \times 4 = 50.28 \text{ cm}^2$ Total surface area of the cap = C.S.A. of the frustum + Area of upper base $= 660 \text{ cm}^2 + 50.28 \text{ cm}^2 = 710.28 \text{ cm}^2$
Question 4.   A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of ₹ 20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 8 per 100 cm2. (Take π = 3.14)   **Solution:**   Radius of the lower end (r_1) = 8 cm   Radius of the upper end (r_2) = 20 cm   Height of the frustum (h) = 16 cm

$$\text{Volume of the container (frustum)} = \frac{\pi h}{3}\left[r_1^2 + r_2^2 + r_1 r_2\right]$$

$$= 3.14 \times \frac{16}{3}\,[8^2 + 20^2 + 8 \times 20]\ \text{cm}^3$$

$$= 3.14 \times \frac{16}{3}\,[64 + 400 + 160]\ \text{cm}^3 = 3.14 \times \frac{16}{3} \times 624\ \text{cm}^3$$

$$= 10449.92\ \text{cm}^3$$

$$= 10.449\ \text{litres} \qquad \left[\begin{array}{l} \because 1000\text{l} = 1\ \text{m}^3 \\ \Rightarrow 1000\text{l} = 1000000\ \text{cm}^3 \\ \Rightarrow \frac{1}{1000}\text{l} = 1\ \text{cm}^3 \end{array}\right]$$

$$\text{Cost of 1 litre of the milk} = ₹\,20$$

$$\therefore \quad \text{Total cost} = ₹\,20 \times 10.449 = ₹\,208.98$$

$$\text{Slant height } (l) = \sqrt{(r_2 - r_1)^2 + h^2} = \sqrt{(20-8)^2 + 16^2} = \sqrt{12^2 + 16^2}$$

$$= \sqrt{144 + 256} = \sqrt{400} = 20 \quad \Rightarrow \quad l = 20\ \text{cm}$$

$$\text{C.S.A. of the bucket} = \pi l\,(r_1 + r_2) = 3.14 \times 20\,(20 + 8)\ \text{cm}^2$$

$$= 3.14 \times 20 \times 28\ \text{cm}^2 = 1758.4\ \text{cm}^2$$

$$\text{Base area of the bucket} = \pi r_1^2 = 3.14 \times 8 \times 8\ \text{cm}^2 = 200.96\ \text{cm}^2$$

$$\text{Metal sheet used} = \text{C.S.A. of the frustum} + \text{base area of the bucket}$$

$$= 1758.4\ \text{cm}^2 + 200.96\ \text{cm}^2 = 1959.36\ \text{cm}^2$$

$$\text{Total cost of metal sheet used} = \frac{8 \times 1959.36}{100} = ₹\,156.75$$

Question 5.
A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter 116 cm, find the length of the wire.

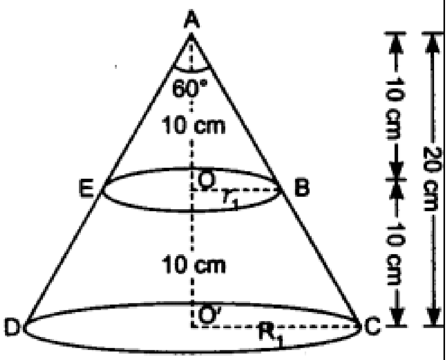
Solution:
Let ADC is a cone with vertical angle 600.
Now, cone is cut into two parts, parallel to its base at height 10 cm.
Radius of larger end of the frustum = R_1

In $\Delta AO'C$, $\tan 30^\circ = \dfrac{O'C}{O'A}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{O'C}{20} = \frac{R_1}{20} \Rightarrow R_1 = \frac{20}{\sqrt{3}} \text{ cm}$$

In ΔAOB, $\tan 30^\circ = \dfrac{r_1}{OA}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{r_1}{10} \Rightarrow r_1 = \frac{10}{\sqrt{3}} \text{ cm}$$



$$\text{Volume of the frustum} = \frac{\pi h}{3}(R_1^2 + r_1^2 + R_1 r_1)$$

$$= \frac{22}{7} \times \frac{10}{3}\left[\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \frac{20 \times 10}{\sqrt{3} \times \sqrt{3}}\right]$$

(where height of frustum $h = 10$ cm)

$$= \frac{22}{7} \times \frac{10}{3}\left[\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right] = \frac{22}{7} \times \frac{10}{3} \times \frac{700}{3} = \frac{22 \times 10 \times 100}{9}$$

A wire be formed having diameter 116 cm and length be H cm

Volume of wire so obtained = $\pi r^2 H$

$$= \frac{22}{7} \times \left(\frac{1}{16 \times 2}\right)^2 \times H = \frac{22}{7} \times \frac{1}{32} \times \frac{1}{32} \times H$$

According to question,

$$\frac{22 \times 10 \times 100}{9} = \frac{22}{7} \times \frac{1}{32} \times \frac{1}{32} \times H$$

$$\frac{22 \times 10 \times 100 \times 32 \times 32 \times 7}{22 \times 9} = H \Rightarrow H = 796444.44 \text{ cm} \Rightarrow H = 7964.4 \text{ m}$$